Spatio-temporal multifractal comparison of 4 rainfall events at various locations: radar data and meso-scale simulations

Presenting author
Auguste Gires

A Gires[1], I. Tchiguirinskaia[1], D. Schertzer[1], S. Lovejoy[2]
[1]*LEESU, Ecole des Ponts ParisTech, France, auguste.gires@leesu.enpc.fr*
[2] *McGill U., Physics dept., Montreal, PQ, Canada*
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1. Introduction

Rainfall intensity is complex not only to analyze and to model but furthermore to measure because it is extremely variable over a wide range of scales in space and time. A common way of representing such variability is to use stochastic multifractals (see Schertzer and Lovejoy, 2011, for a recent review, and Schertzer et al., 2010, for applications to hydrology). It basically relies on the physically based notion of multiplicative cascades. In the specific framework of universal multifractals (UM) only three scale independent parameters are used to quantify the scaling variability of rainfall.

In this paper the validity of a simple space-time scaling model relying on an anisotropy exponent between space and time is investigated through the multifractal analysis of four rainfall events. The theoretical framework of Universal Multifractals is briefly reminded in section 2. The rainfall data (radar data and rainfall outputs of numerical models) is presented in section 3. The results of this comparative analysis are discussed in section 4. Finally it should be mentioned that more details about the multifractal analysis of each event are available in Gires et al., 2011, 2012a and 2012c.

2. Brief reminder of the Universal Multifractal (UM) framework

The Universal Multifractal framework is only briefly presented here. For more details one can refer to the recent review by Schertzer and Lovejoy (2011). Let us denote R_λ a field at a resolution λ (=L/l, the ratio between the outer scale and the observation scale). If the field exhibits a scaling behaviour, then its power spectra is fully characterized with the help of a spectral slope β:

$$E(k) \approx k^{-\beta} \quad \text{Eq. 1}$$

If the field is multifractal, then the statistical moment of an arbitrary order q scales as:

$$< {R_\lambda}^q > \approx \lambda^{K(q)} \quad \text{Eq. 2}$$

Where $K(q)$ is the scaling moment function and fully characterizes the variability through scales of the field. In the specific framework of Universal Multifractals $K(q)$ depends of only three scale invariant parameters:

- H the non-conservation parameter (H=0 for a conservative field).
- C_1, the mean intermittency co-dimension, which measures the clustering (intermittency) of the (average) intensity at smaller and smaller scales (C_1=0 for a homogeneous field).
- α, the multifractality index ($0 \le \alpha \le 2$), which measures the clustering variability with respect to intensity level.

In that case we have:

$$K(q) = \frac{C_1}{\alpha - 1}\left(q^\alpha - q\right) + Hq \quad \text{Eq. 3}$$

UM parameters are estimated with the help of the Double Trace Moment technique (Lavallée et al., 1993).

3. Presentation of the rainfall data

3.1 5-9 September 2005, South-East of France

The first studied event is heavy rainfall episode (known as “Cevenol”) that occurred in the South-East of France on 5-9th September 2005. Two types of data are used:

- Radar data: an area of size 512 km x 512 km during 5 days is extracted from the Météo-France radar mosaic. The Météo-France processing includes correction of ground clutter, partial beam blocking and vertical profile of reflectivity effects (Tabary, 2007; Tabary et al., 2007). The resolution of the data is 1 km in space and 15 min in time. The total rainfall depth during 16h starting 5th September at 2pm is displayed Fig. 1.a. The temporal evolution of the average rainfall rate is plotted Fig. 1.b. There are 2.5 h of missing data during the second and main peak after roughly 30h.
- Méso-NH simulations: the rainfall output of numerical simulations of the same event performed with the help of Meso-NH, a non-hydrostatic mesoscale research model developed by Météo-France and Laboratoire d’Aérologie (Univ. Paul Sabatier, Toulouse, France) (Lafore et al., 1998), is also analysed. The resolution of the data is roughly 3 km in space and 15 min in time. There are seven consecutive simulations lasting 18 h starting every 12h. As it is visible on Fig. 1.b the model

does not generate any rainfall at the beginning of each simulation. Therefore the first two hours of each simulation are always neglected.

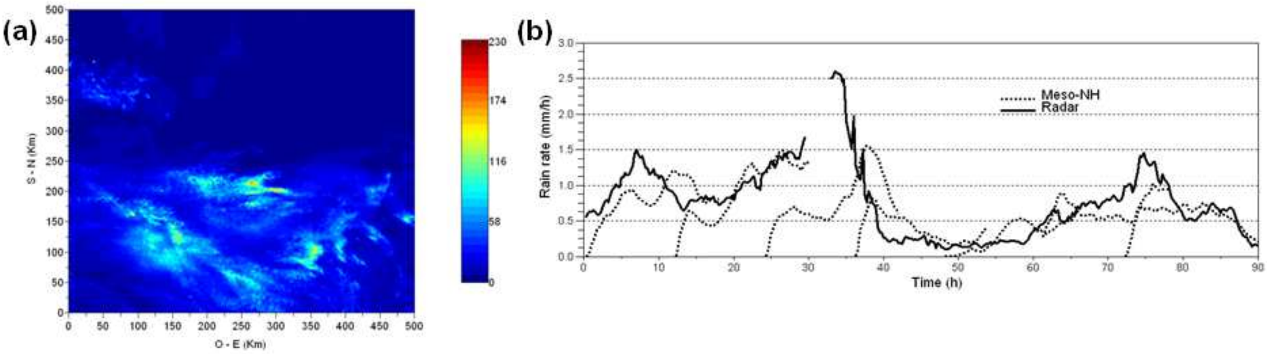

Figure 1: (a) Total rainfall depth in mm via radar data during 16h starting 5th September 2005 at 2pm. The coordinates (in °, Réseau géodésique français 1993) of the four corners are 46.3-1.3, 41.5-1.3, 46.2-8.1 et 41.4-7.5. (b) Temporal evolution of average rain rate over the studied area for the two data types

3.2 9th February 2009, Paris area

The second studied event occurred on 9th February 2009 over the Paris area. The data comes from the C-band radar of Trappes which is operated by Météo-France. An area of size 256 km x 256 km during 13h is studied. The total rainfall depth ranges from 0 to 27 mm. The resolution of the data is 1 km in space and 5 min in time.

3.3 9th February 2009 and 7th July 2009, London area

The two last rainfall events analysed in this paper occurred on 9th February 2009 and 7th July 2009 over the London area. The data comes from the Nimrod mosaics (Harrison et al. 2000), a radar product of the UK Meteorological Office. Areas of size 64 km x 64 km during 21h are analyzed. The total rainfall depths for both events are displayed Fig. 2. More localized rainfall cells are visible for July event.

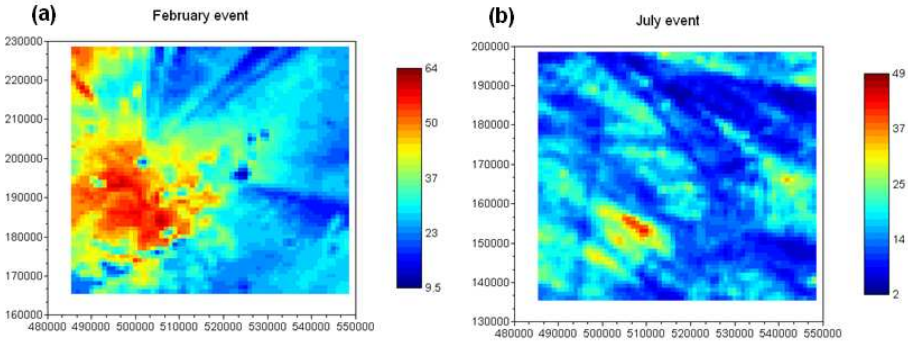

Figure 2: Map of the total rainfall depth (mm) of the studied area over the London area for the February (left) and the July (right) events. The coordinate system is the British National Grid (units: m).

4. Results and discussion

4.1 Spatial analysis of the rainfall event

In this section we present the results of a multifractal analysis performed on ensemble average for each event, i.e. all the time steps (a 2D map of rain rate) are upscaled independently and taken into account in Eq. 2. Some results with a separate analysis for each time step are available in Gires et al. 2011.

Fig. 3 displays the power spectra for the Cevenol and the 9th February 2009 Paris rainfall events. The spectral slope is a first confirmation of the scaling behaviour with nevertheless a break. For the Cevenol event (Fig. 3.a) this break occurs for wave number between 25 and 35 which corresponds to distances between 14 km and 20 km. A spectral slope is also visible for the Paris event (Fig. 3.b), and the break is less obvious. However we still considered one for k=16 (corresponding to 16 km) to be consistent with the multifractal analysis (see below) where it is clearly visible. The estimated spectral slopes for the four events are reported in Table 1. Whatever the event and the range of scales, they are smaller than the dimension of the embedding space (2 here), which means that the multifractal analysis can be carried out directly on the rain rate field and one does not need to consider the fluctuations of the field.

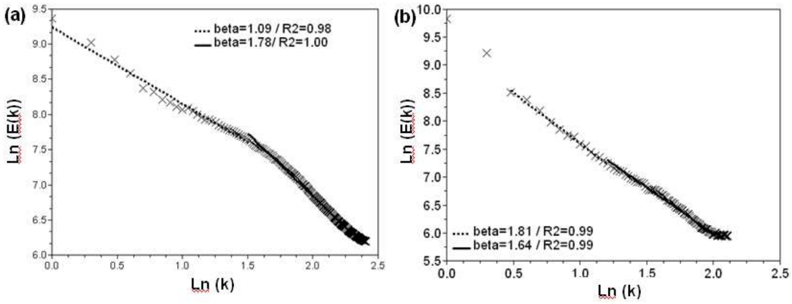

Figure 3: Spectra for the Cevenol event (a) and the 9th February 2009 Paris event (b)

Event	Range of scales	α	C_1	H	β
5-9th Sept 2005, South-East of France	Small scales : 1 - 16 km	1.62	0.16	0.53	1.78
	Large scales : 16 - 512km	0.89	0.45	0.34	1.09
9th February 2009 Paris area	Small scales : 1 - 16 km	1.52	0.056	0.37	1.64
	Large scales : 16 - 256km	1.08	0.28	0.61	1.81
9th February 2009 London area	1-64 km	1.62	0.14	0.56	1.87
7th July 2009 London area	1-64 km	0.92	0.49	0.57	1.49

Table 1: Estimates of the UM parameters α, C_1 and H for the different studied events

Eq. 2 in a log-log plot for the 9th February 2009 Paris and 7th July 2009 London rainfall events are displayed in Fig. 4. The straight lines (or portion of it) reflect the scaling behaviour of the rainfall fields (on a limited range of scales). For the Paris event there is a break at roughly 16 km. This visual insight is confirmed by the coefficient of determination R^2 of the linear regression which is equal (on average on the plotted moments) to 0.96 for the range of scales 1-16 km and 0.91 on the range of scales 16-256 km, whereas it is equal to 0.91 if no break is taken into account. A break was also observed on the Cevenol event on both the radar data (at 16 km) and the Meso-NH simulations (at 20-25 km). Such break is also reported for other rainfall events in the Paris area (Tchguirinskaia et al., 2011). The fact that this break is visible for various events and data types suggests that it is not an artefact, and that it is likely to be physical caused. For the Cevenol event an interpretation with the topography would not be valid. Indeed a multifractal analysis on the topography data of the area does not reveal any break at least on scales ranging from 2-3 km to 150-200 km. The scale of the break of 16 km reminds the size of the classical notion of rain cell. A possible interpretation (Gires et al., 2012d) could be a misrepresentation of the rainfall zeros (i.e. a pixel with no measured rainfall) in this simple multifractal framework. It should be noted that these two explanations are not contradictory.

Concerning the rainfall events in London, there does not seem to be a break. For instance the scaling curve (Eq. 2) of the July event is plotted in Fig. 4.b and does not exhibit any. The mean coefficient of determination of the linear regressions is equal to 0.98 without taking any break into account on the whole range of scales 1-64 km (it is also equal to 0.98 for the February event). Nevertheless this statement should be qualified in so far as the available range of scales (1-64 km) is not wide enough to reveal a break as for the French events. Furthermore a slight curvature is visible on the scaling curves and could correspond to a break. Data on a wider range of scales would be needed to confirm that.

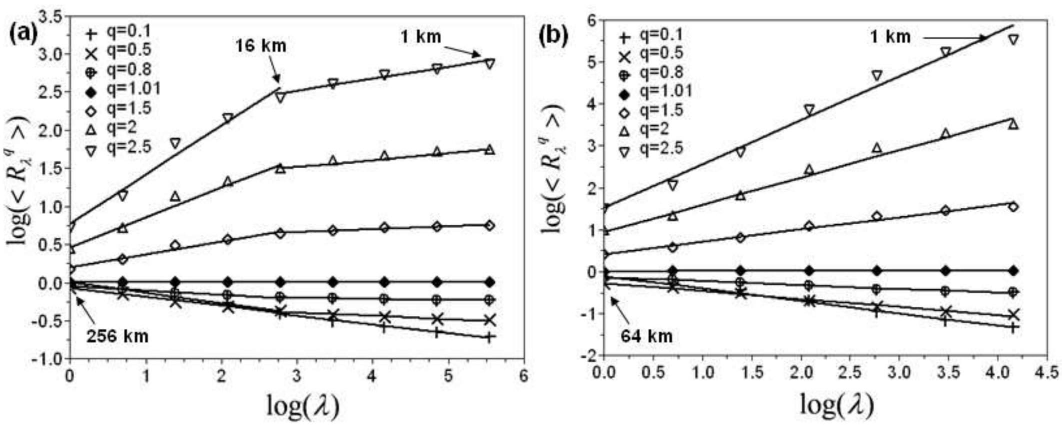

Figure 4: Illustration of the scaling behaviour (Eq. 2) for the Paris event (a) and the 7th July 2009 London one (b)

The estimates of the UM parameters are reported in Table 1. As most of the estimates available in the literature they do not take into account of a potential bias introduced by the numerous zeros, which would affect all the studied events (see Gires et al., 2012b for more details about that). Concerning the small scales, it appears that the estimates for the two French events and the 9th February 2009 London one (where the range of scales is slightly greater) are comparable i.e. $\alpha \sim 1.5 - 1.7$, $C_1 \sim 0.05 - 0.2$ and $H \sim 0.3 - 0.6$. These values are also rather similar to the ones found by Tessier et al. (1993) on radar reflectivity or Verrier et al. (2010) on radar images of African monsoon. The similarity between the estimates despite very different meteorological situations seems to confirm the robustness of the theoretical framework and to highlight a possible universality of the UM parameters values for rainfall. These values are also in agreement with the multifractal parameters characterizing the atmospheric turbulence (Lazarev et al. 1994; Schmitt et al., 1992), which would mean that rainfall behaves as a passive scalar for small scales.

For large scales we find smaller α and greater C_1. For the Cevenol event, we even find $\alpha<1$ which reflects a significant statistical change with regards to small scales. Indeed $\alpha>1$ corresponds to the non-classical notion of "self organized criticality" (Bak et al., 1988) which implies that the singularities are no longer bounded. This deep difference in behaviour between scales implies that the small scale statistics cannot always be deduced from large scale ones. It should be noted that the large scale parameters are closer the values classically found for low resolution (usually hourly or daily) long (few months or years) time series from rain gauges, i.e. $\alpha \sim 0.5 - 0.7$, $C_1 \sim 0.3 - 0.5$ (de Lima and de Lima, 2009; de Lima and Grasman, 1999; Fraedrich and Larnder, 1993; Ladoy et al., 1993; Olsson, 1995; Tessier et al., 1996).

Concerning the 7th July 2009 rainfall event in London, we find α=0.92 and C_1=0.45 which is quite different from the other events. As it can be seen on Fig. 2 this event seems to be characterized by rainfall cells of smaller size (few km) that the others. A possible interpretation of the differences in the parameters is that this event appears at the scales 1-64 km as a small scale model of the other events at 16-256 km. Thus the small scale UM parameters of this event are rather similar to the large scale ones of the others. For that matter the UM parameters estimated only on the range of scales 1-4 km (which is not relevant given the available data as we have seen) are α=1.16 and C_1=0.28 which is closer to the small scale parameters of the other events. Smaller scale data for this event would be needed to confirm this interpretation, along with multifractal analysis of other rainfall events.

.4.2 Spatio-temporal analysis

In the previous section, we performed spatial analysis, i.e. each 2D map of a time step is considered as an independent sample and independently upscaled in Eq. 2 (see fig. 5.a for an illustration). It is also possible to do temporal analysis where samples are not 2D maps, but pixel time series (see fig. 5.b). UM parameters of spatial and temporal analysis can be related in the framework of a scaling spatio-temporal model. The simplest one for rainfall (Deidda, 2000; Macor, 2007; Marsan et al., 1996; Radkevich et al., 2008) relies on an anisotropy exponent between space and time. Thus the scaling moment functions in space and time should be proportional, that is to say:

$$K_{space}(q) = \frac{K_{time}(q)}{1-H_t} \quad \text{Eq. 4}$$

Where H_t is the anisotropy exponent between space and time. This implies identical α and that the ratios between the C_1 and the H are the same:

$$\frac{C_{1,space}}{C_{1,time}} = \frac{H_{space}}{H_{time}} = \frac{1}{1-H_t} \quad \text{Eq. 5}$$

The estimates of the UM parameters for the large scales of the Cevenol event and both data types (radar and Meso-NH simulation outputs) are reported Table 2. We will not mention here the small scale results because the available range of scales (15min-1h) in time is too narrow to provide accurate estimates. Nevertheless, few comments can be found in Gires et al. (2011). These large scale UM parameters estimates are in overall agreement with the spatio-temporal theoretical framework, especially for radar data. Indeed the α are rather similar, the ratio of $C_{1,space}/C_{1,time}$ found corresponds to an anisotropy exponent of 0.22, and the ratio H_{space}/H_{time} leads to H_t equal to 0.38. These values are rather comparable and compatible with the theoretical value of 1/3 which would correspond to the Kolmogorov theory (Kolmogorov, 1962; Marsan et al., 1996) assuming that rain cells have the same lifetime like eddies. For Meso-NH simulation outputs the ratio of C_1 leads to H_t equal to 0.30, which is in agreement with the theoretical framework. On the other hand the ratio of H does not fit with this framework since it leads to H_t =-0.01.

To confirm these results spatio-temporal analysis are also performed (see Fig. 5.c for an illustration). In that case space and time are considered at once in the upscaling of the field. More precisely, when the spatial resolution is divided by λ_{xy} then the temporal one is divided by $\lambda_t = \lambda_{xy}^{\ 1-Ht}$. Here we have H_t =1/3 and we choose λ_{xy}=3 and λ_t=2 ($3^{2/3}$~2.08). The results for the Cevenol event of this spatio-temporal analysis are displayed Table 2. Concerning the large scales of the radar data, the same parameters as in the spatial analysis are retrieved which is consistent with the theoretical framework. For the Méso-NH simulations we find a α greater than in the spatial analysis, and a C_1 estimate between the spatial and the temporal one.

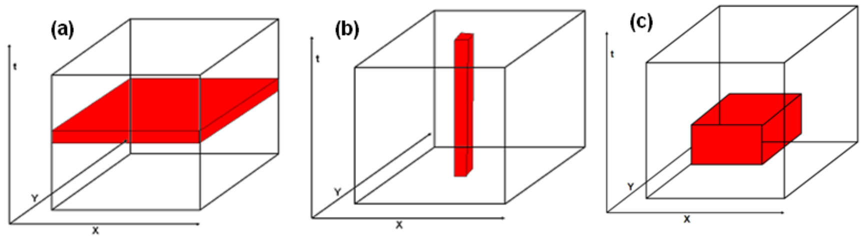

Figure 5: Illustration of the independent sample considered in spatial (a), temporal (b), and spatio-temporal (c) analysis.

	Radar data				Méso-NH simulations			
Type of analysis	Range of scales	α	C_1	H	Range of scales	α	C_1	H
Spatial analysis	16 – 512 km	0.89	0.45	0.34	22 – 720 km	0.54	0.71	0.36
Temporal analysis	1 – 16 h	0.82	0.35	0.21	1 – 16 h	0.54	0.50	0.37
Spatio-temporal analysis	20 km - 486 km 1 h – 8 h	0.87	0.42	-	28 km - 680 km 1 h – 8 h	0.68	0.47	-

Table 2: Estimates of UM parameters α, C_1 and H for the Cevenol event of 5-9 September 2005 in the South of France (radar data and Meso-NH simulations)

Finally let us mention that this type of analysis was also implemented on the two London events. The estimated UM parameters are in Table 3. For the February event the α for the spatial and temporal analysis are close and the ratio of C_1 corresponds to an underlying H_t equal to 0.28, which is quite close from the theoretical value of 1/3. On the other hand the ration of H leads to H_t equal to 0.68 which is not in agreement with the theoretical framework. It should be mentioned that the estimates of H are less reliable than the ones of C_1 and α because of difficulties in the estimation of spectral slopes. Nevertheless it remains acceptable to validate this simple spatio-temporal framework for this event. Concerning the July event the ratios of C_1 and H lead to H_t respectively equal to 0.02 and 0.82. For this event without being excluded this spatio-temporal framework is not explicitly validated.

Type of analysis Range of scales	Event	α	C_1	H
Spatial analysis 1 – 64 km	9th February 2009	1.62	0.14	0.56
	7th July 2009	0.92	0.49	0.57
Temporal analysis 5 min – 11 h	9th February 2009	1.52	0.10	0.21
	7th July 2009	0.72	0.48	0.10

Table 3: Estimates of the UM parameters α, C_1 and H for the two event of the London area and for different type of analysis.

.5. Conclusion

In this paper the spatio-temporal variability of rainfall is analyzed for 4 events (a Cevenol event in the South of France, one in the Paris area and two in the London area) with the help of the Universal Multifractals. In this framework the variability is characterized by only 3 scale invariant parameters. Radar data is used, plus Meso-NH simulation outputs for the Cevenol event.

The main conclusions, which are an important step in the validation of spatio-temporal scaling models and multifractal simulations of rainfall fields, are:

- The scaling behaviour of the rainfall field is confirmed, with a break observed at roughly 20 km in the spatial analysis for most of the studied event. A possible interpretation in a misrepresentation of the numerous zeros in this simple framework.
- The comparison of the UM parameters in spatial and temporal analysis are in overall agreement with the simple space-time scaling model that relies on an anisotropy exponent between space and time. This is confirmed by direct spatio-temporal analysis of the rainfall fields
- These results hints at a possible universality of the UM parameters for rainfall fields: $\alpha \sim 1.5 – 1.7$, $C_1 \sim 0.05 – 0.2$ and $H \sim 0.3 – 0.6$.

Concerning the Meso-NH simulation, it appears that the rainfall outputs exhibit similar qualitative behaviour, i.e. a scaling behaviour with a break at roughly 20-25 km and an agreement (less good than with the radar data, especially with regards to the non-conservation parameter *H*) with the spatio-temporal unified framework. On the other hand the numerical estimates of the UM parameters are significantly different from the radar ones: the variability is under-represented in Meso-NH, whereas the mean intermittency is over-represented.

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