$Ib_\lambda^{(q)} \equiv < Z_{e,\lambda}{}^q > / < Z_\lambda{}^q >$

GETTING HIGHER RESOLUTION RAINFALL ESTIMATES: X-BAND RADAR TECHNOLOGY AND MULTIFRACTAL INSIGHTS

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INTRODUCTION

Weather radars remain the only measuring devices that provide space-time estimates of rainfall. However, the classical rainfall products with pixels of 1 km2 do not meet the relevant scales of urban hydrology. Urbanisation sprawling induces considerable changes in landscape and land-use, and requires a more detailed observation and integrated predictions of the water balance. A key factor of this subtle balance is the extreme variability of the rain field from planetary scales to centimetre scales, which corresponds to the fact that the rainrate is strongly scale dependent. This behaviour results from the fact that rain accumulation is a (mathematical) singular measure [1]. The latter property has many important and practical consequences, especially for small scale observations.

This question is particularly important due to a recent breakthrough in radar technology (see e.g. [2]): dual polarimetry allows to introduce a radar self-calibration with the help of an estimate of size distribution based on differential reflectivity due to the drop flattening. This is rather indispensable to remove the measurement bias induced by rain attenuation, which increases with the radar frequency. The X-band radars became therefore usable not only for rainfall detection, but although for rainfall measurements. There are several projects, including one in the Paris region, that are focussed on getting higher resolution rainfall estimates with X-band radars. They call our attention on the need to improve the present retrieval schemes of rainrate from reflectivity and to consider more realistic assumptions that those which are currently used.

RADAR REFLECTIVITY AND RAINFALL VARIABILITY

Most of the developments on polarimetric retrieval schemes have been focused on getting observables less dependent on the drop size distribution (DSD) variability, and based on the differential reflectivity Z_{DR} and the specific differential phase shift K_{DP}, in order to get robust correction schemes for the absorption. The basic problem of the "speckle" effect or "drop rearrangement" remains rather untouched. It results from the fact that the ubiquitous hypothesis of an homogeneous distribution of the drops is physically implausible, although mathematically convenient to factorize drop distribution into a Poisson distribution of centres and a translation invariant DSD.

The impact of the speckle effect was tentatively estimated by [3] with the help of a comparison of the effective reflectivity field Z_e, which is the Fourier transform of the backscatter distribution, and the traditional radar reflectivity factor $Z\lambda$:

$$Z_{e,\lambda} = \left| \int_{B_\lambda} \sigma_\Lambda(\mathbf{x}) e^{i\mathbf{k}_r \cdot \mathbf{x}} d\mathbf{x} \right|^2 \qquad Z_\lambda = \int_{B_\lambda} \sigma_\Lambda(\mathbf{x})^2 d\mathbf{x} \qquad (1)$$

Here $\lambda = L/\ell$ is the non-dimensional radar resolution, i.e. the scale ratio of a given outer scale L with respect to the measurement scale ℓ, here the radar pulse length that is typically ℓ=100*m*-1*km* and $B\lambda$ is the corresponding radar volume; the resolution $\Lambda = L/\eta$ corresponds to the inner scale η of the backscatters/rainfield variability and corresponds to discrete integration.

$\sigma_\Lambda(x) = v(x)/vol(B_\lambda)$ is the relative volume of the drop centred at the non-dimensional location $x=r/L$ and of volume $v(x)$; k_r is the non-dimensional radar pulse wave-number (i.e. also a-dimensionalized by L). The classical radar model corresponds to incoherent small-scale variability (homogeneous distribution of backscatters): the backscatter phases in Eq. (2) are independent identically distributed random variables and therefore for a large number of drops, the cross terms cancel leading to: $Z_{e,\lambda} \approx Z_\lambda$.

The deviations of

$$Ib_\lambda^{(q)} \equiv < Z_{e,\lambda}{}^q > / < Z_\lambda{}^q > \qquad (2)$$

with respect to the unity are measures of the speckle bias (<.> denotes the ensemble average) of the corresponding statistical moment of order q.

The classical *Z-R* relationship for incoherent scattering $(Z = aR^b)$ can be obtained [4] by considering:

$$Z \propto \int dD\, D^6 N(D) \qquad R \propto \int dD\, D^3 v(D) N(D) \qquad (3)$$

The terminal velocity $v(D)$ is assumed to be scaling with drop size diameter D.

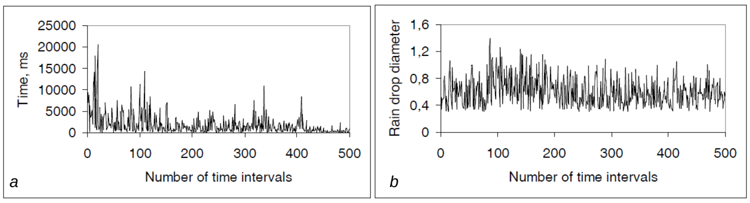

Fig. 1 *a-b*: Example of an infrared OSP record of time intervals (*a*) between each of recorded drops with their corresponding diameters (*b*).

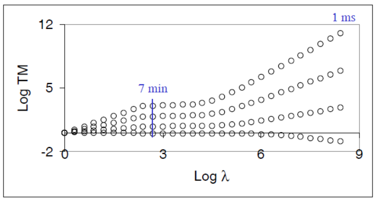

Fig. 2: The statistical moments estimated on OSP rainfall volume exhibit two clear scaling regions: from 1*ms* to about 2*s* and from 7 *min* up to 1 day (total sample time).

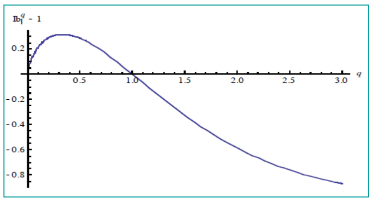

Fig. 3: Estimate of the relative speckle bias *Ib* for the statistical moment of order *q*. One may note that *Ib* is already of 33% and 60% for *q*=1.5;2 respectively.

SPECKLE AND MULTIFRACTALITY

The radar reflectivity $Z\lambda$ and $R\lambda$ are expected to be multifractal as the rainrate is [5]. This is furthermore supported by the analysis of a high resolution time series of rain drop diameters and time intervals obtained with the help of an infrared optical spectro-pluviometer (OSP), see Figs.1-2. In particular, the statistical moments exhibit multifractality, i.e. for any positive order q:

$$\left\langle \sigma_\lambda^q \right\rangle = \lambda^{K_\sigma(q)} \left\langle \sigma_1^q \right\rangle \qquad (4)$$

where $K_\sigma(q)$ is the scaling function of the moments. With the help of the technique of "normalized power densities" [6] that yields:

$$\left\langle \left(\int_{B_\lambda} \sigma_\Lambda{}^\eta dx \right)^q \right\rangle \propto \lambda^{K_\sigma(q,\eta)}; \qquad K_\sigma(q,p) \equiv K_\sigma(q,p) - qK_\sigma(p)$$

one obtains from Eqs. (1), (3) and (4):

$$\begin{aligned} &< Z_\lambda{}^q > \propto \lambda^{K_Z(q)}; && K_Z(q) = K_\sigma(q,2) \\ &< R_\lambda{}^q > \propto \lambda^{K_R(q)}; && K_R(q) = K_\sigma(q,2/b) \\ &< Z_{e,\lambda}{}^q > \propto k^{K_Z(q)}; && K_Z(q) = K_\sigma(q,2) \end{aligned} \qquad (5)$$

and therefore Eq. (2) becomes:

$$Ib_\lambda^{(q)} \equiv < Z_{e,\lambda}{}^q > / < Z_\lambda{}^q > \propto (k/\lambda)^{K_Z(q)} \qquad (6)$$

To obtain a more precise idea of this bias, let consider that the backscatter field is a universal multifractal field [7] for which :

$$K(q,p) = \frac{C_1}{\alpha - 1} p^\alpha (q^\alpha - q); \qquad K(q) \equiv K(q,1)$$

The codimension of the mean field C_1 measures the mean intermittency and the multifractality index α measures how the intermittency varies with various orders of statistical moments. The following estimates obtained for the multifractal distribution of raindrop volumes in time [8]: $C_{1,\sigma} \approx 0.35; \alpha_\sigma \approx 0.82$

yield the corresponding graph for $Ib_\lambda^{(q)} - 1$ displayed in Fig. 3, for $k_r/\lambda \approx 3$ by considering the wave-length of an X-band radar ($k_r{}^{-1} \approx 3cm$ and the inter-drop distance $\lambda^{-1} \approx 1cm$).

DISCUSSION OF THE RESULTS

Higher resolution rainfall data are potentially available with the help of polarimetric X-band radars. However, in spite of the increasing sophistication of the retrieval algorithms, the speckle problem remains rather unsolved. Using a multifractal approach, we obtain results that tend to show that it has more important impacts than usually considered. Indeed, according to the estimates obtained above the speckle bias is already of 33% and 60% for the estimates of the statistical moments of order q =1.5; 2 respectively, contrary to a global estimate of 25% [6]. Although, a more detailed analysis is required, the present results tends to show that there is a necessity to work directly on the effective reflectivity than on the standard reflectivity.

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