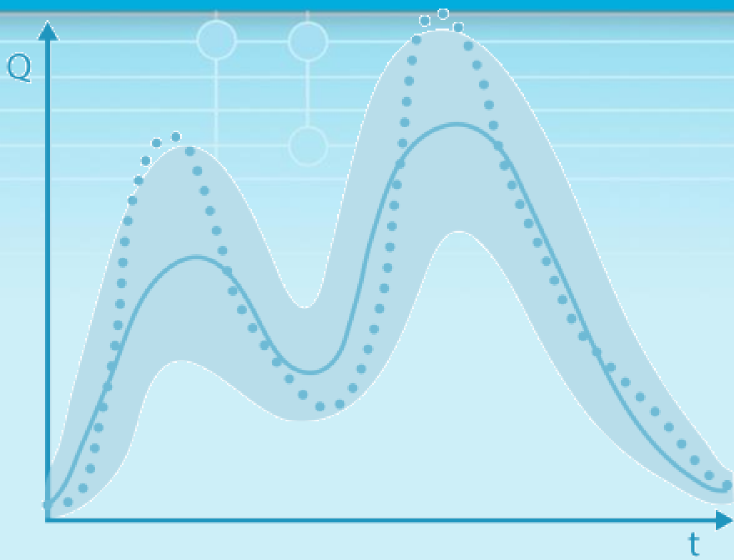
How can statistics help us to get reliable predictions despite model bias?



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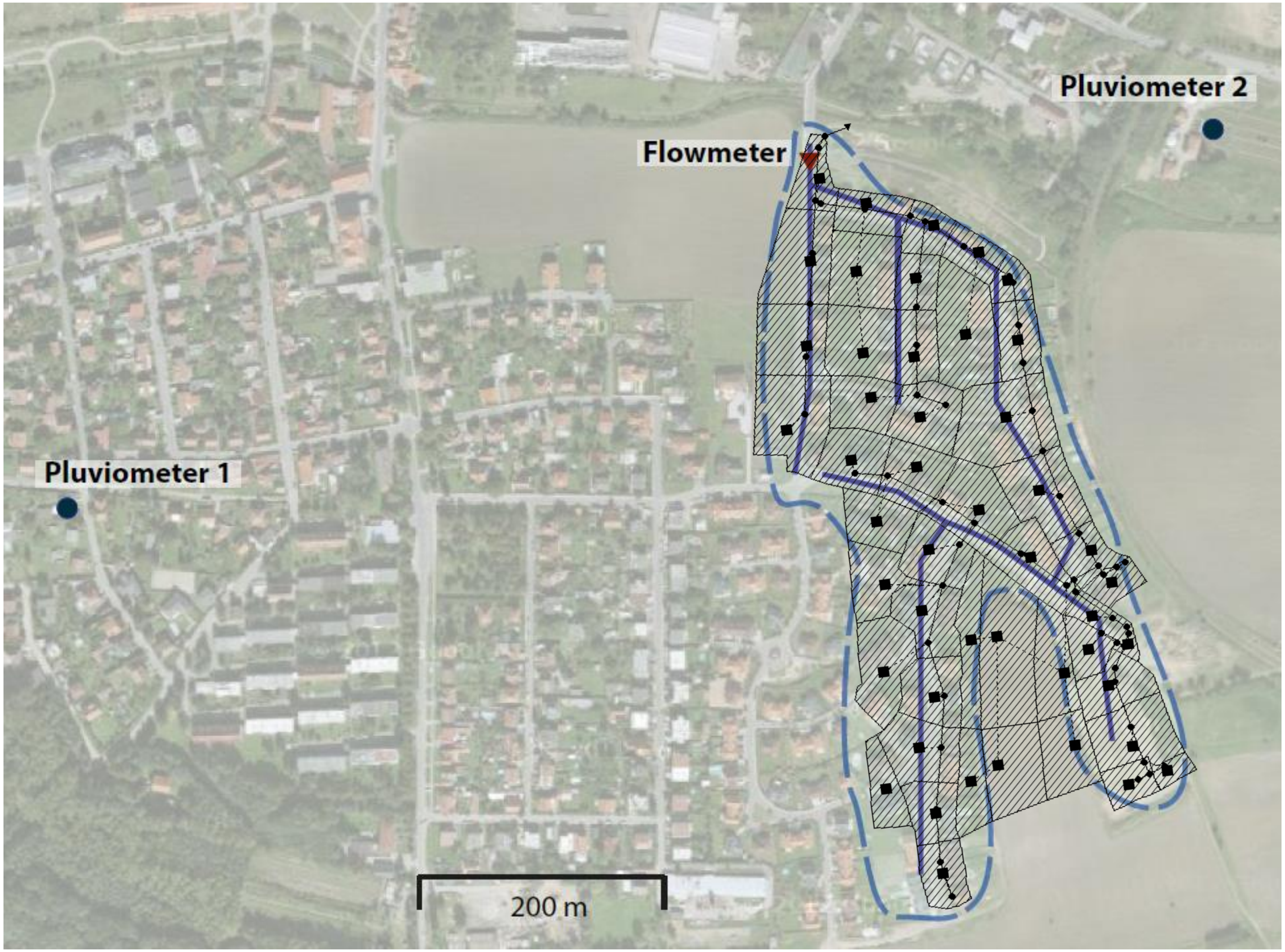
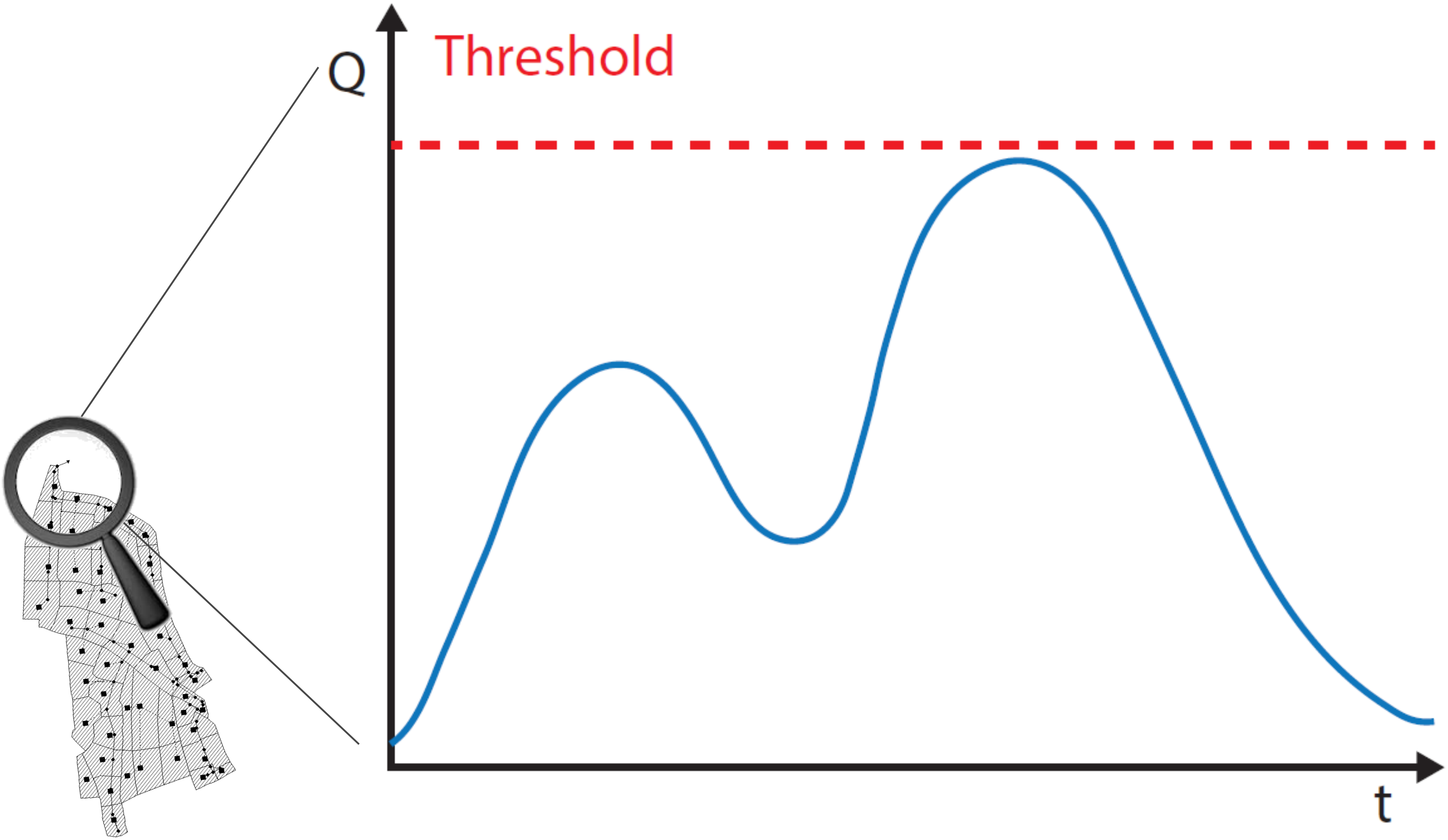
Swiss Federal Institute of Technology Zurich

eawag aquatic research

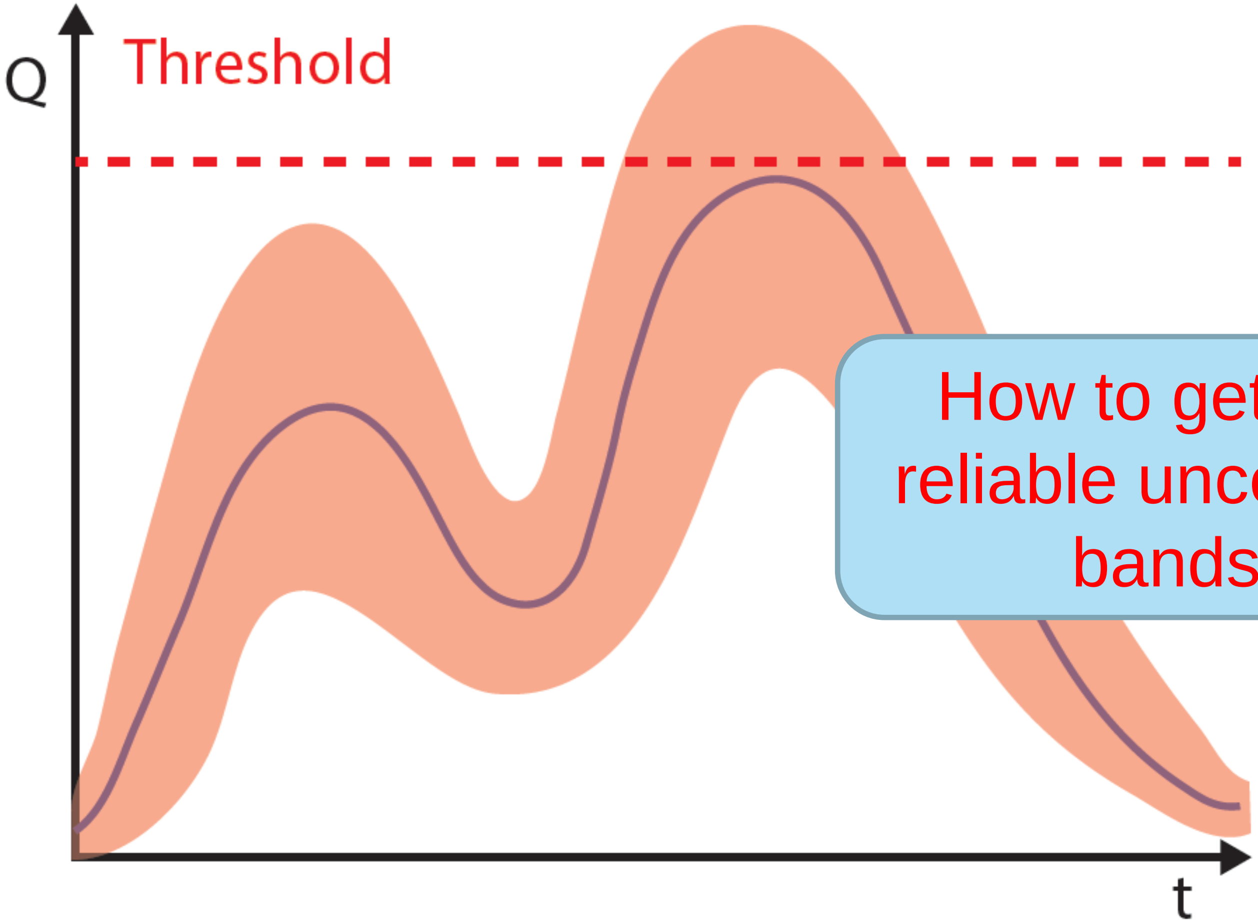
Our typical situation



We want to predict the output

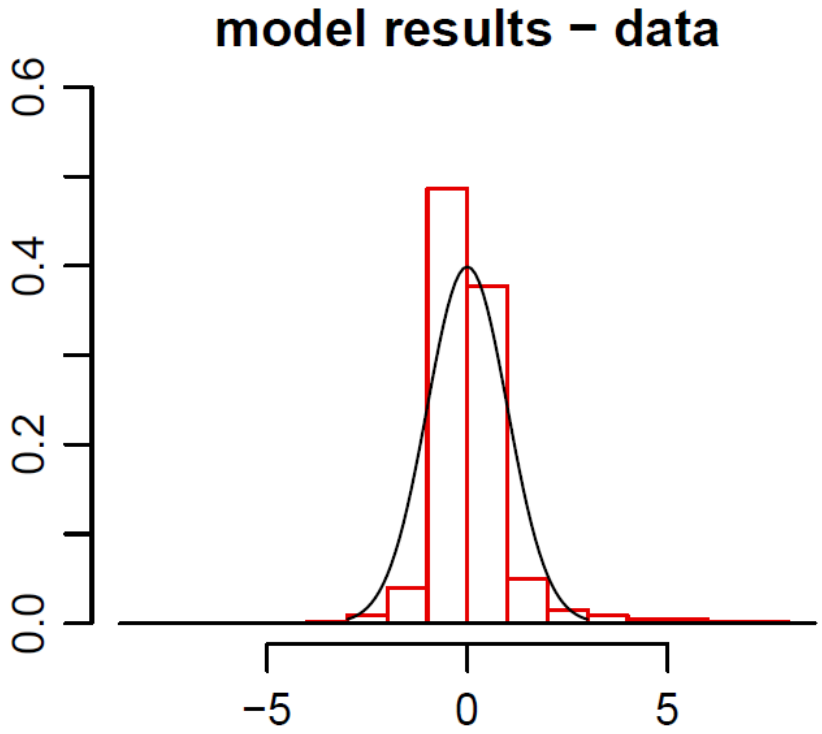


A line is not enough…



How to get such reliable uncertainty bands?

Statistics can come in handy for realistic predictions



Likelihood function

$$f(\boldsymbol{y}_o \mid \boldsymbol{\theta}, \boldsymbol{\psi}, \boldsymbol{x}) = \frac{(2\pi)^{-\frac{n}{2}}}{\sqrt{\det(\boldsymbol{\Sigma}(\boldsymbol{\psi}, \boldsymbol{x}))}}$$

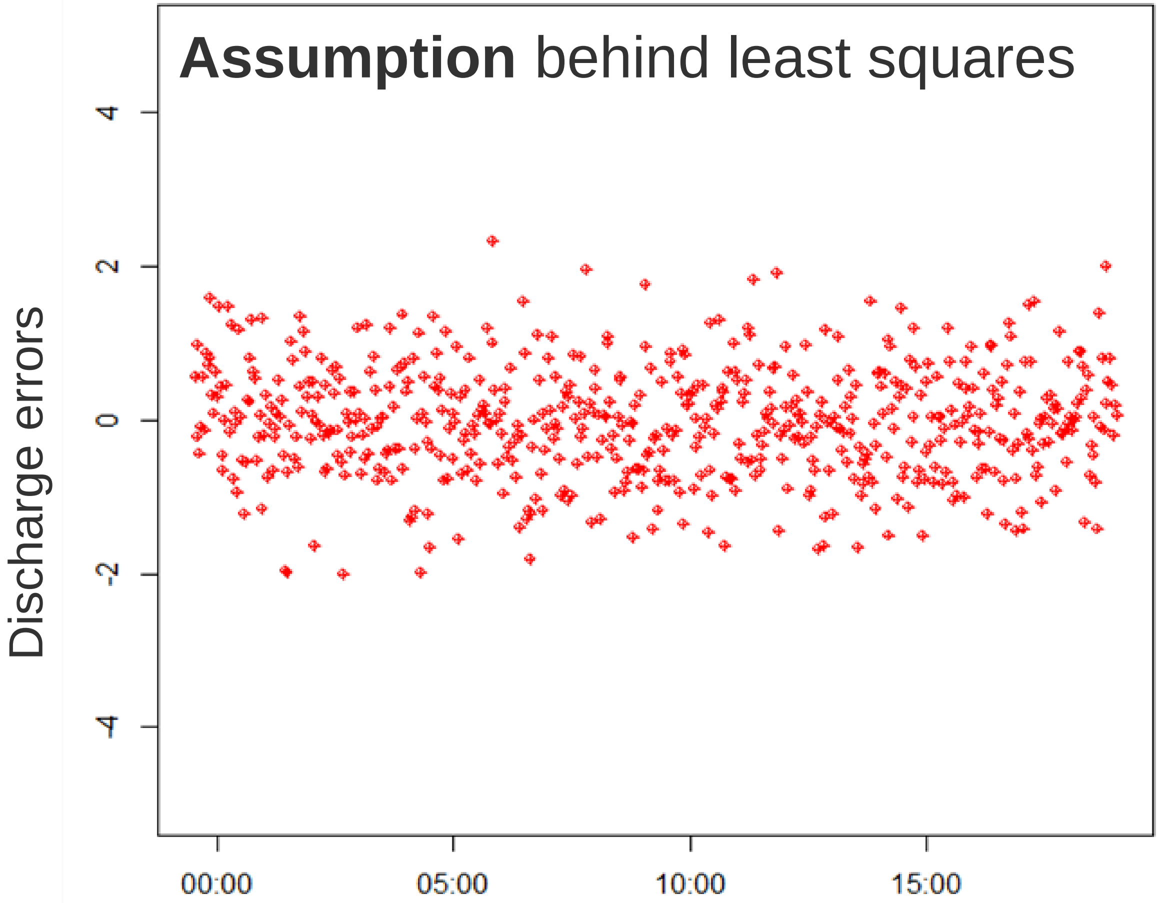
$$\cdot \exp\left(-\frac{1}{2}\left[\tilde{\boldsymbol{y}}_o - \tilde{\boldsymbol{y}}_{\mathrm{M}}(\boldsymbol{\theta}, \boldsymbol{x})\right]^{\mathrm{T}} \boldsymbol{\Sigma}(\boldsymbol{\psi}, \boldsymbol{x})^{-1} \left[\tilde{\boldsymbol{y}}_o - \tilde{\boldsymbol{y}}_{\mathrm{M}}(\boldsymbol{\theta}, \boldsymbol{x})\right]\right) \prod_{i=1}^{n} \frac{\mathrm{d}g}{\mathrm{d}y}(y_{o,i}, \boldsymbol{\psi})$$

Statistically-based doesn’t always mean reliable

To work, uncertainty analysis has to be based on a **realistic likelihood** function!

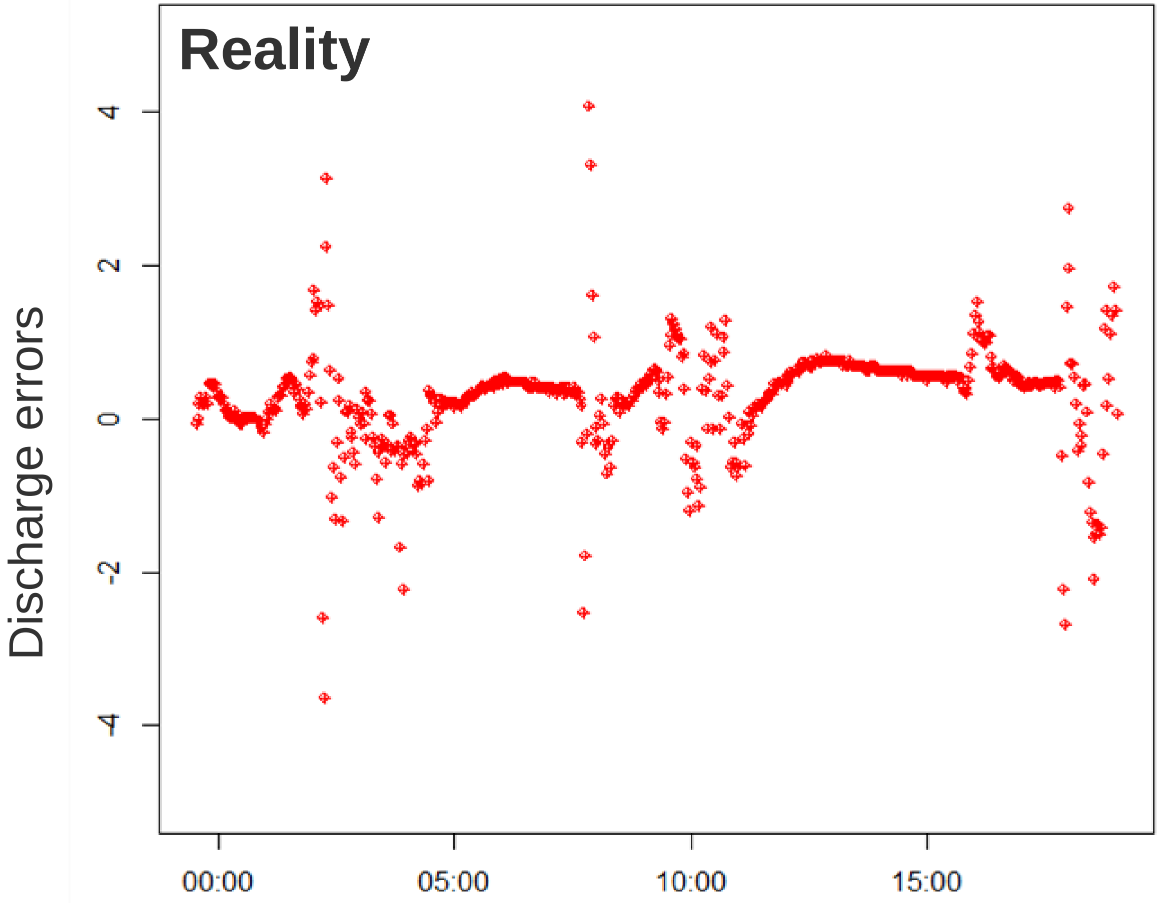
Traditional uncertainty assessment is overly idealized

model results - data = random uncorrelated errors

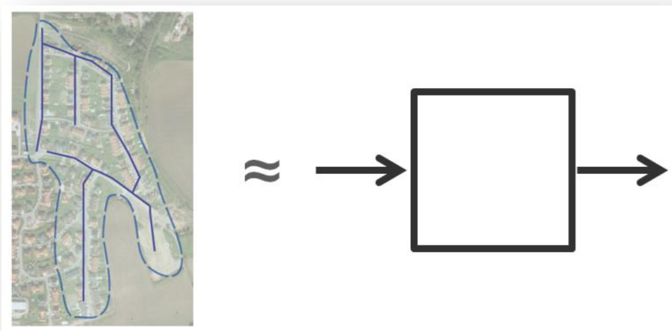


Reliable uncertainty assessment should account for reality

model results - data = non-stationary systematic errors

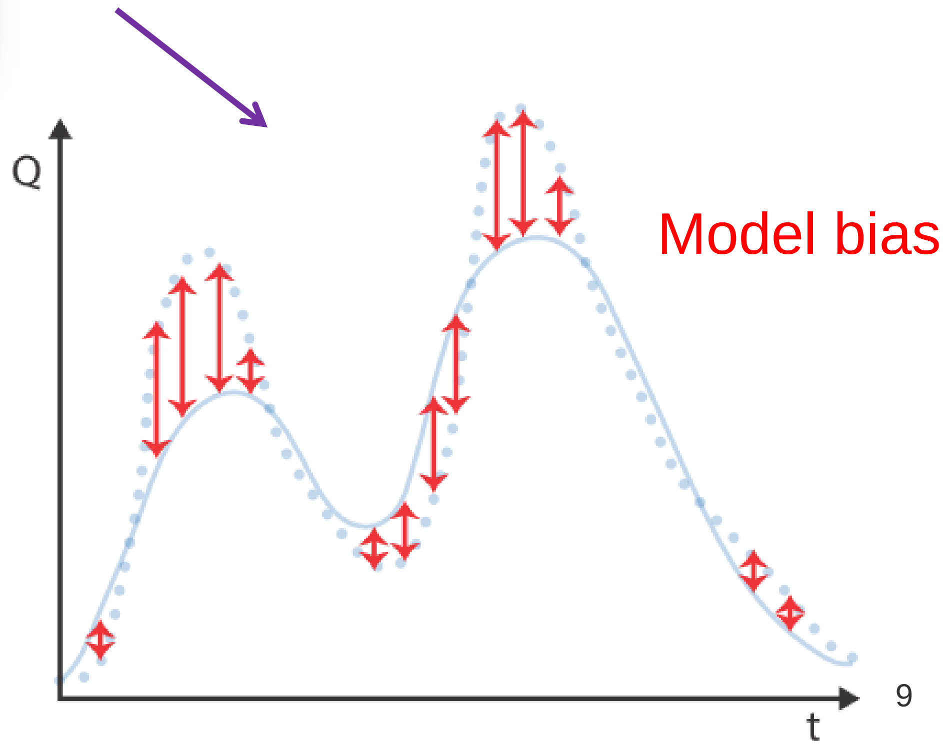


Why do we have these systematic deviations?



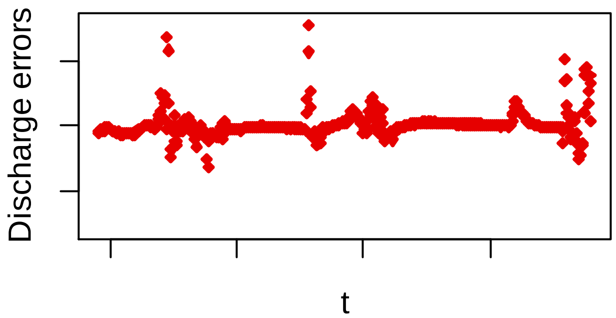
Structural deficits

Input errors



Real errors are better approximated by a bias description

We model the systematic output errors as a rainfall dependent Gauss-Markov process



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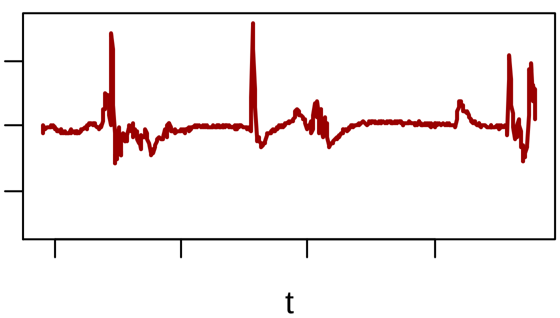
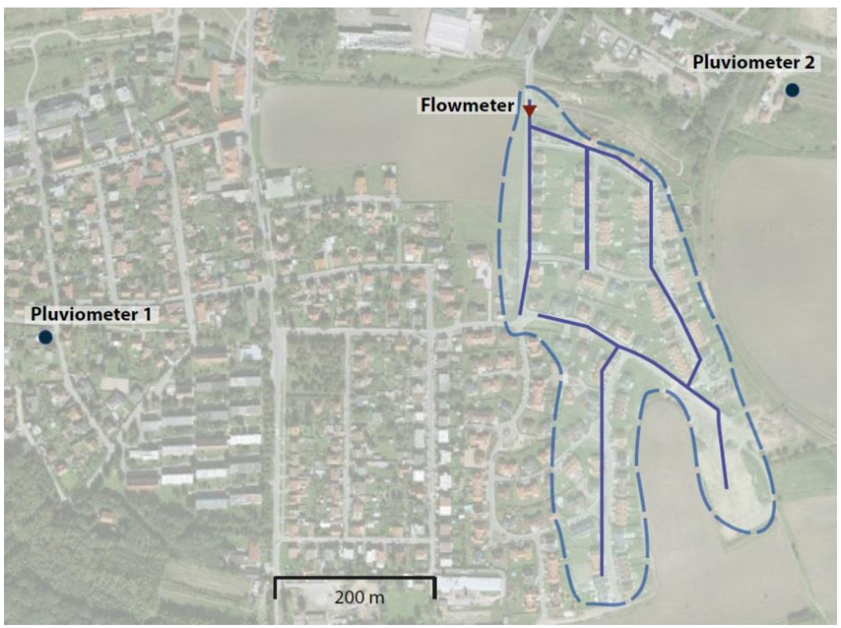
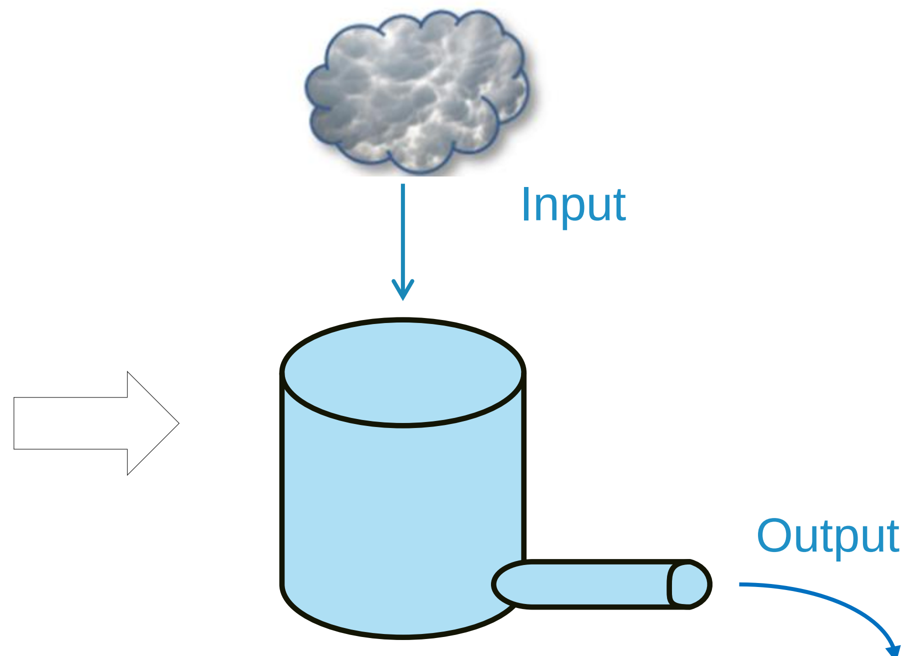


$$\mathrm{d}B_{\mathrm{M}}(t) = -\frac{B_{\mathrm{M}}(t)}{\tau}\mathrm{d}t + \sqrt{\frac{2}{\tau}\left(\sigma_{B_{\mathrm{ct}}}^2 + (\kappa x(t-d))^2\right)}\mathrm{d}W(t)$$

How does it work?

Example: flow simulations over 2 days



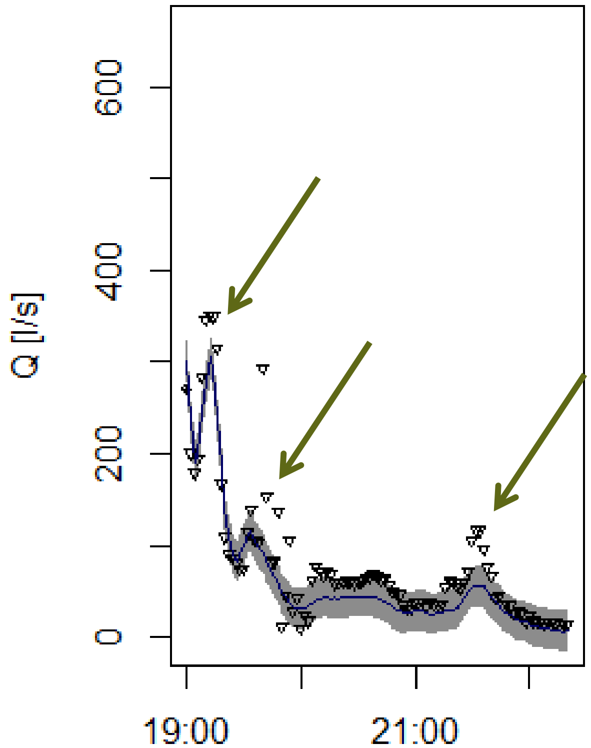


Stormwater catchment, CZ

Hydrologic model

Results: Hindcasting

Ignoring bias



Del Giudice et al., Improving uncertainty estimation (2013) - HESS

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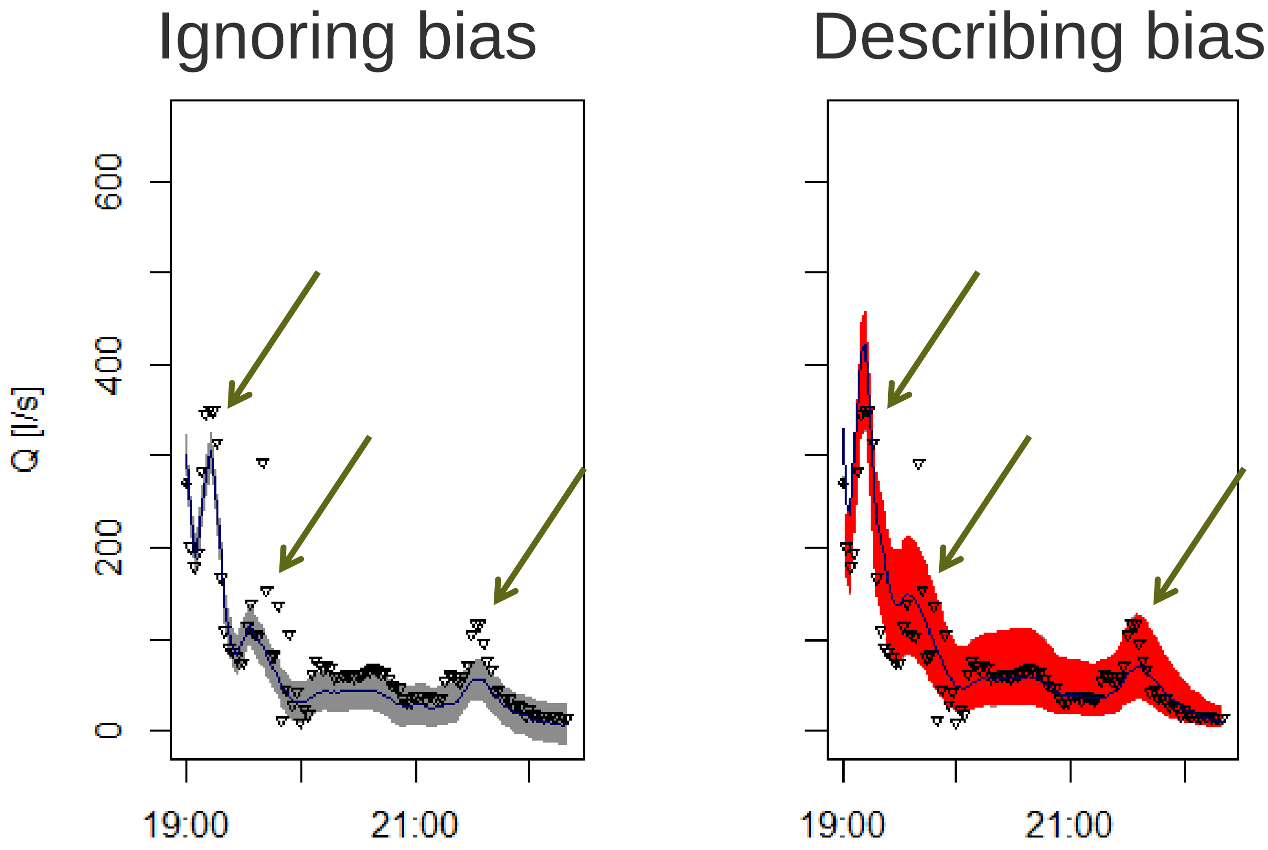

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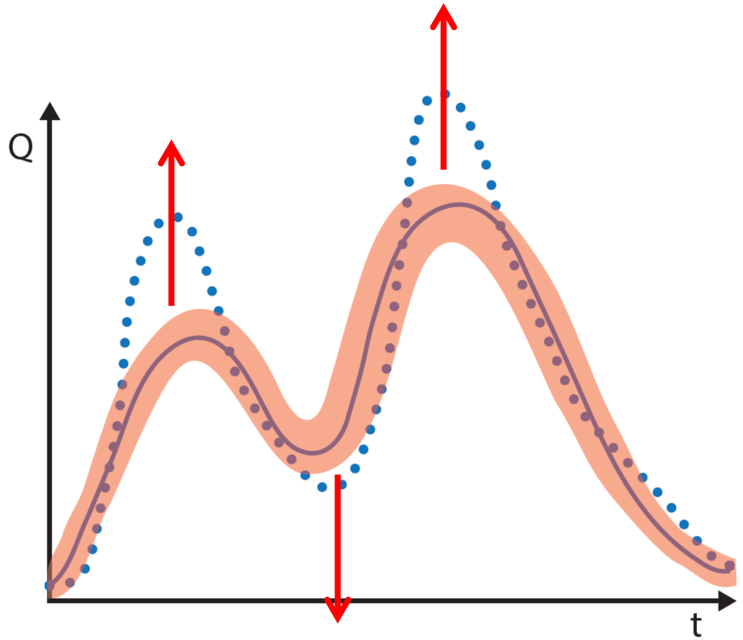
Discussions (I)

Rainfall & structural errors matter: with the bias description predictions are more reliable and robust



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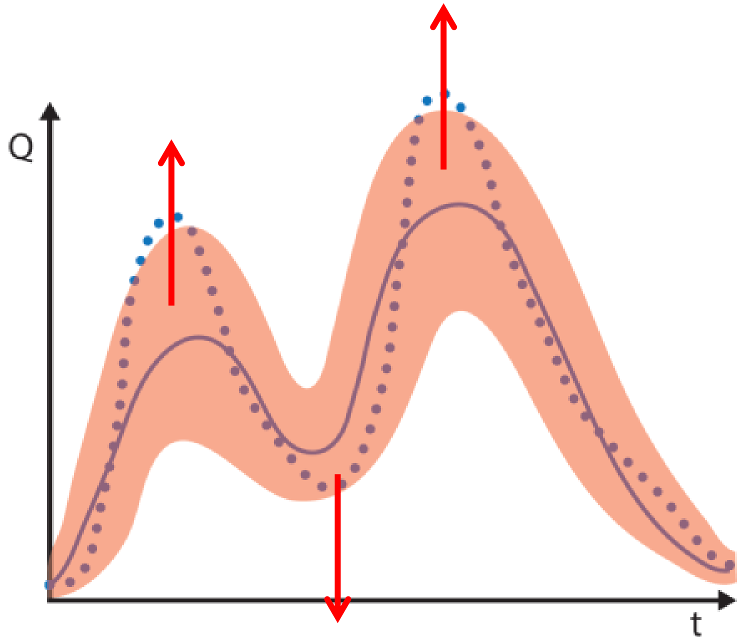
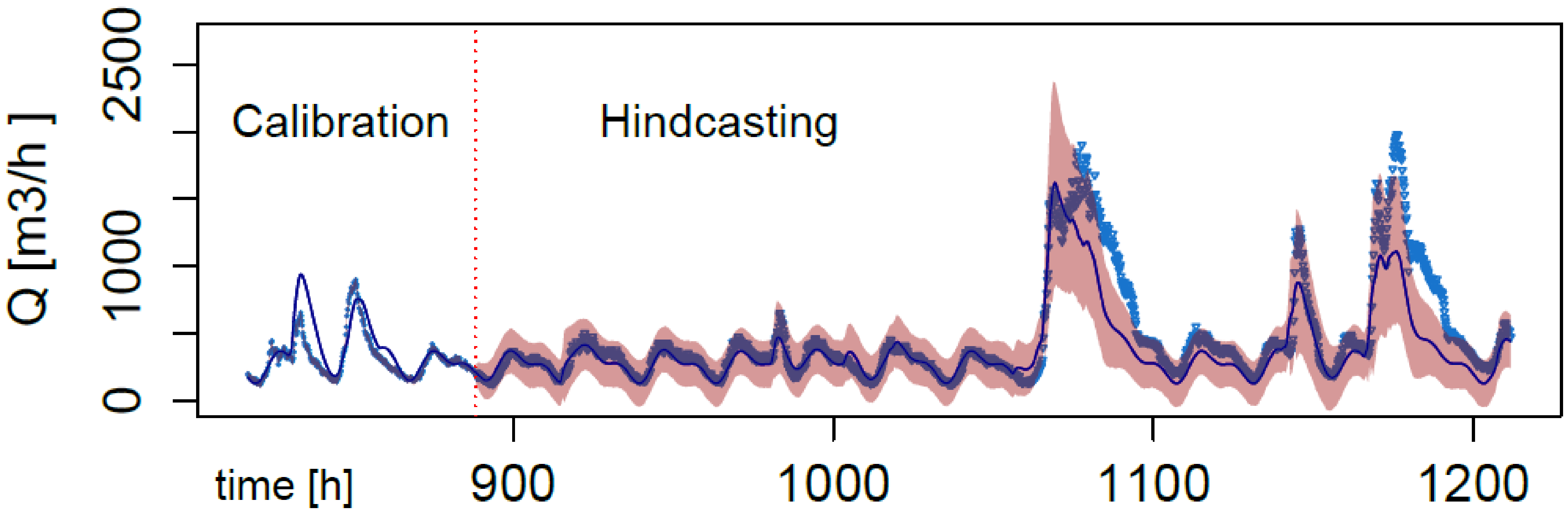


Discussions (II)

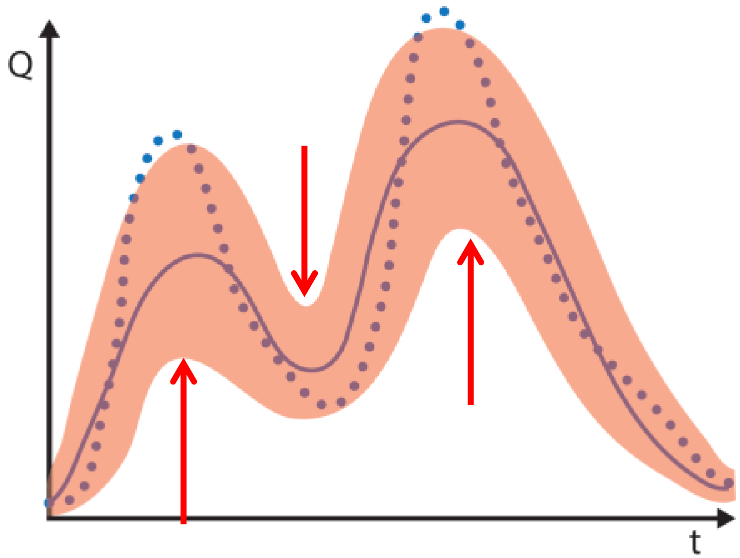
Current implementation is optimal for long-term predictions with parsimonious models



Del Giudice, Mikkelsen et al., Comparing two stochastic techniques (in prep.) - WRR

Discussions (III)

Bias description only improves the assessment of the uncertainties but doesn't reduce them



Del Giudice et al., Model bias and complexity (2014) – EM&S

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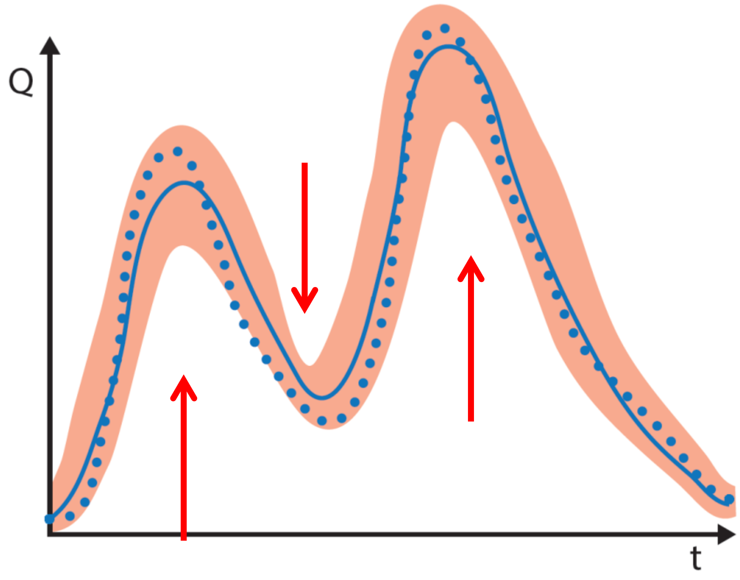

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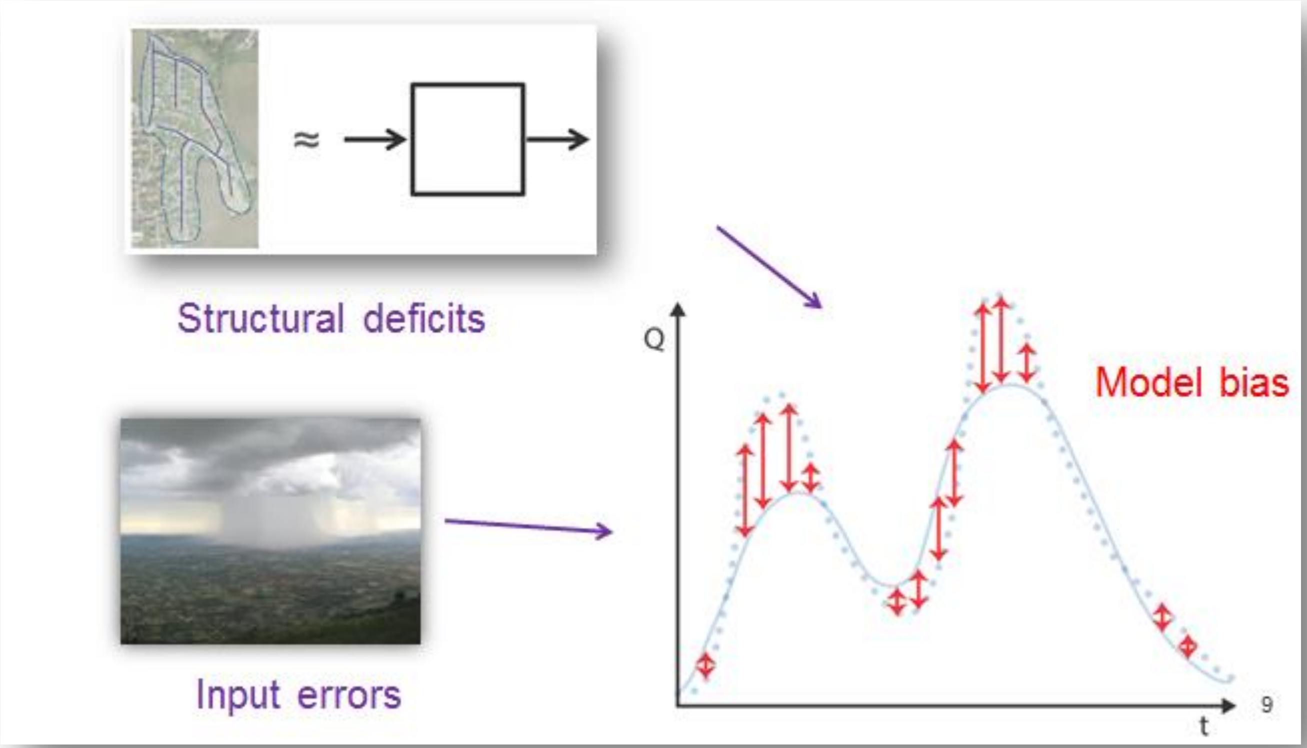
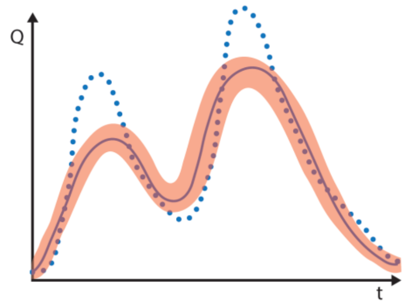
Current challenge: quantify and reduce input uncertainty



Conclusions

i. Rainfall/structural errors play a crucial role in urban drainage predictions



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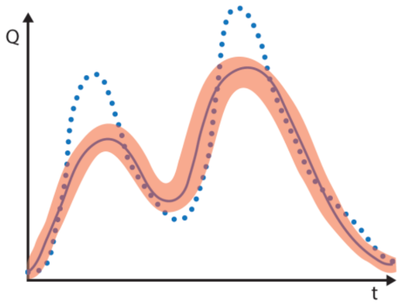
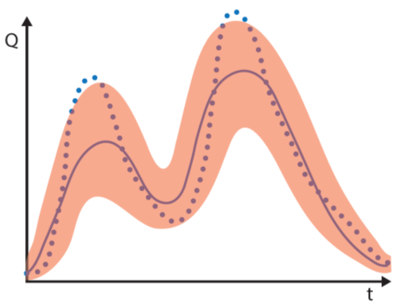


ii. A reliable way to quantify these uncertainties is the bias description



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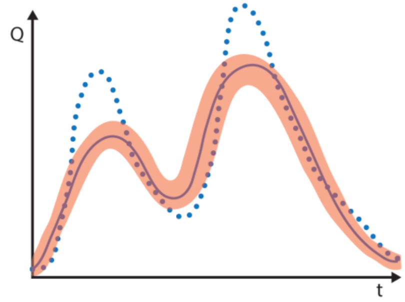

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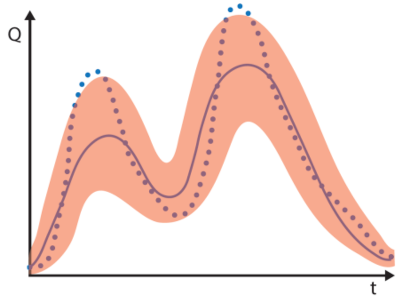

iii. A more advanced input description will minimize rainfall uncertainties