Flood modelling uncertainty analysis by variance decomposition

Patrick Willems, KU Leuven

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Model uncertainty estimation methods

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- classical Bayesian approaches
- pseudo Bayesian methods such as GLUE
- recursive model and parameter identification techniques
- methods based on frequentist statistical inference
- …

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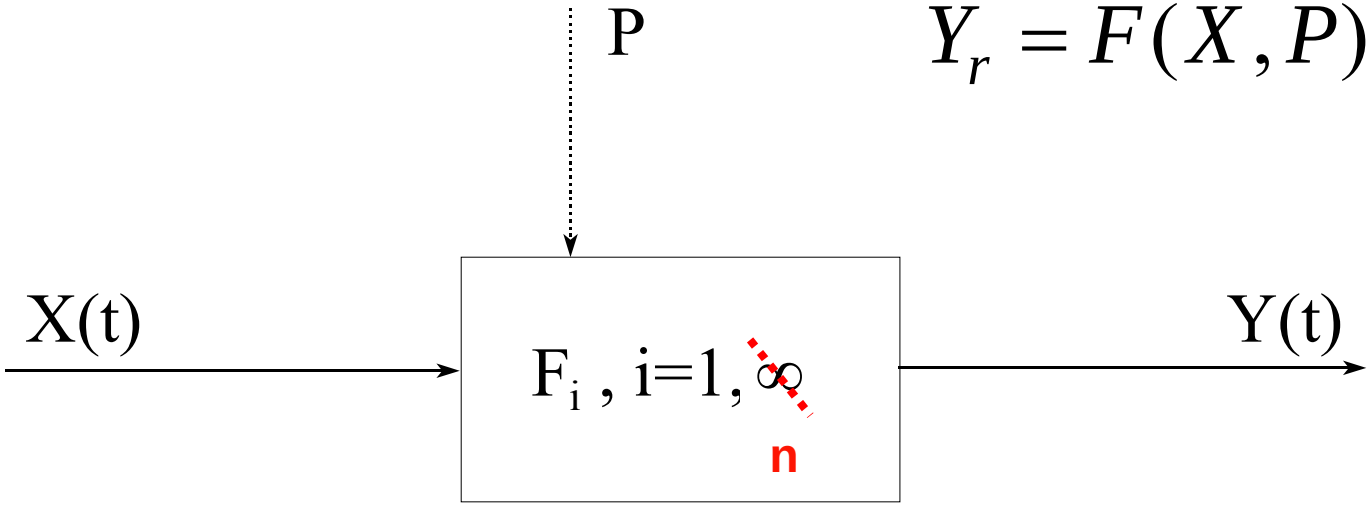
Model uncertainty estimation methods

Under the frequentist statistical inference paradigm:

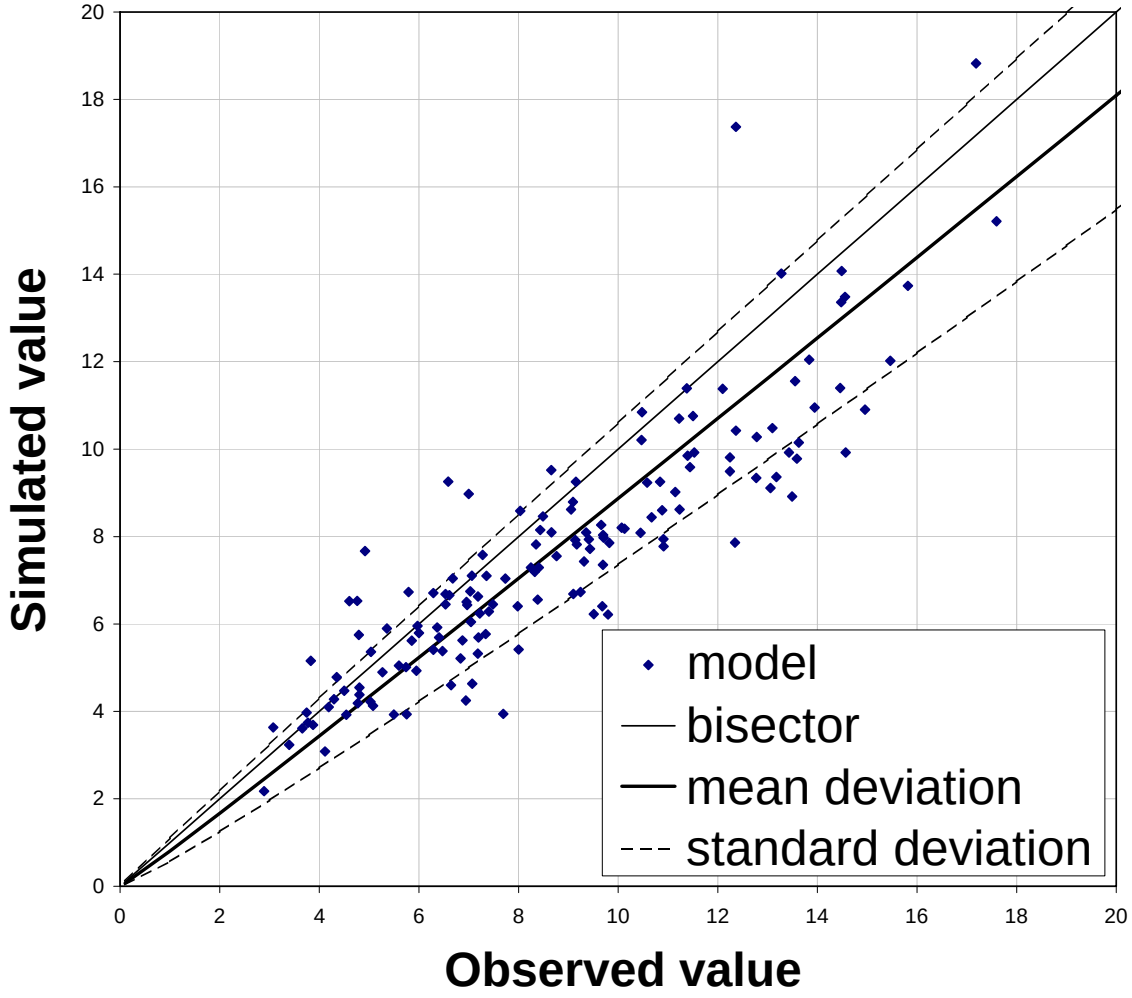
$$Y_r = F(X, P) + E_Y$$



Real system

Modelled system

Total uncertainty quantification



Model residuals:

$$E_Y = Y_m - Y_o$$

Heteroscedasticity!!

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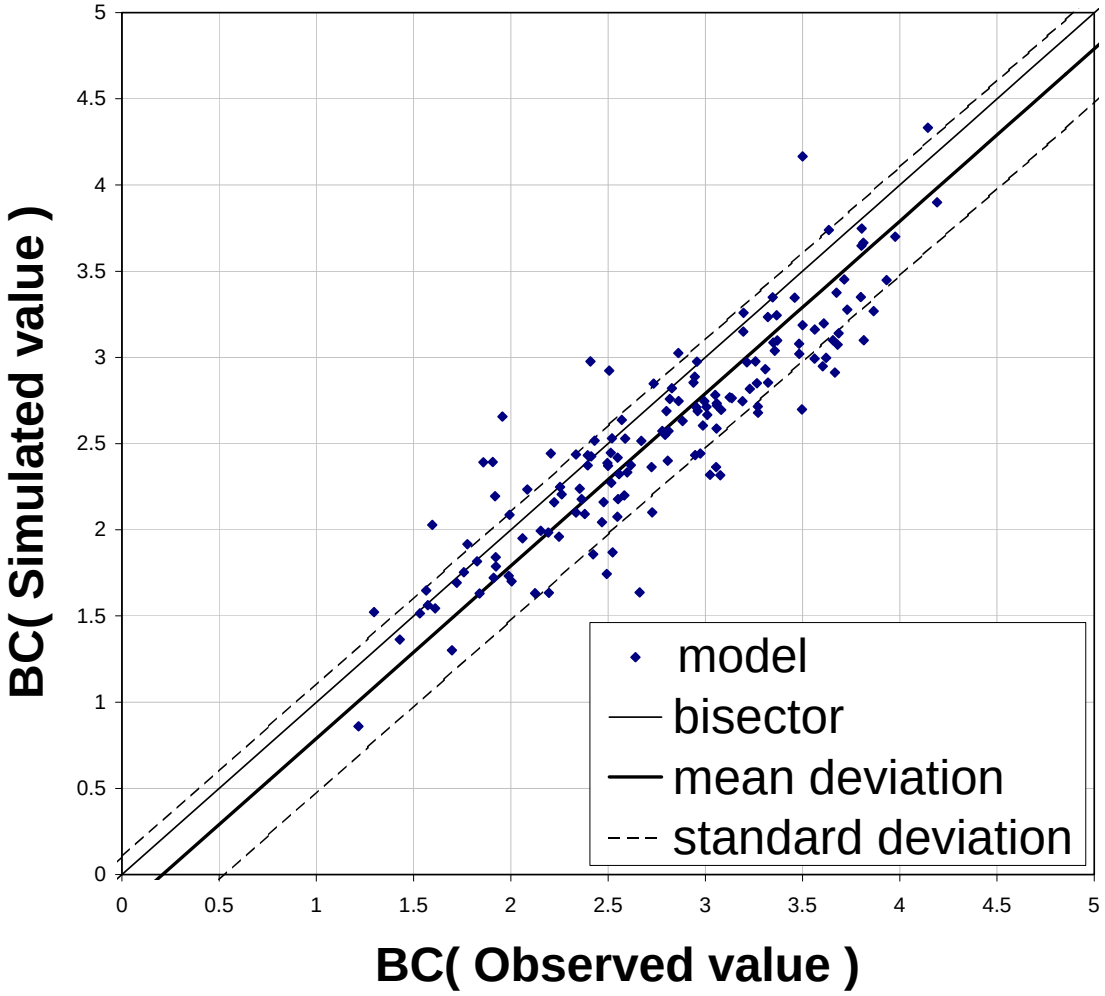
Willems P. (2009), 'A time series tool to support the multi-criteria performance evaluation o
Environmental Modelling & Software, 24(3), 311-321

Total uncertainty quantification



Box-Cox transformation to reach **homoscedastic model residuals**:

$$BC(q) = \frac{q^{\lambda} - 1}{\lambda}$$

$$0 < \lambda \leq 1$$

$$\lambda = 0 : BC = \log$$

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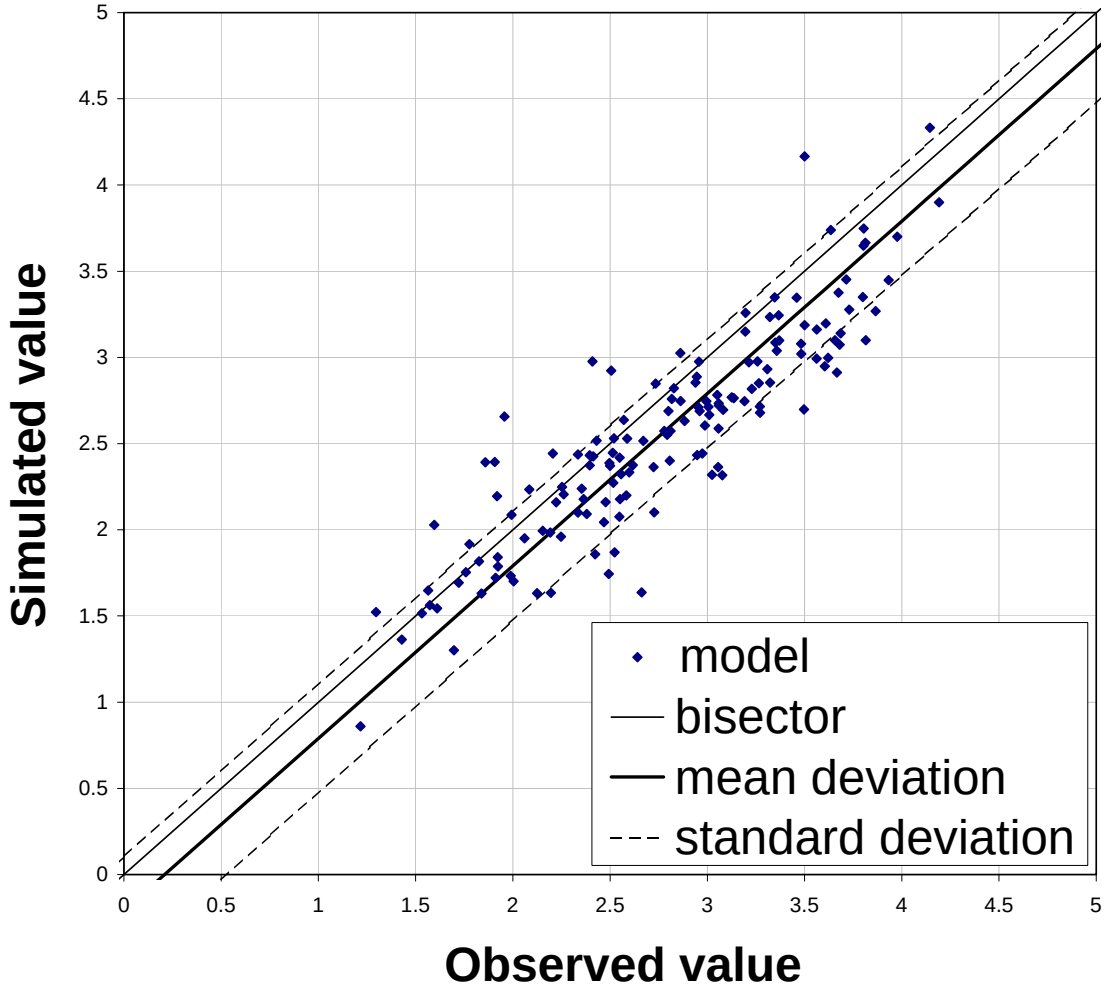

Model residuals:

$$E_Q = Q_m - Q_o$$

Mean model residual (**model bias**):

$$ME = \overline{E}_Q = \sum_{i=1}^{n} \frac{E_Q(i)}{n}$$

Model residual variance:

$$S_{E_Q}^2 = \sum_{i=1}^{n} \frac{(E_Q(i) - \overline{E}_Q)^2}{n-1}$$

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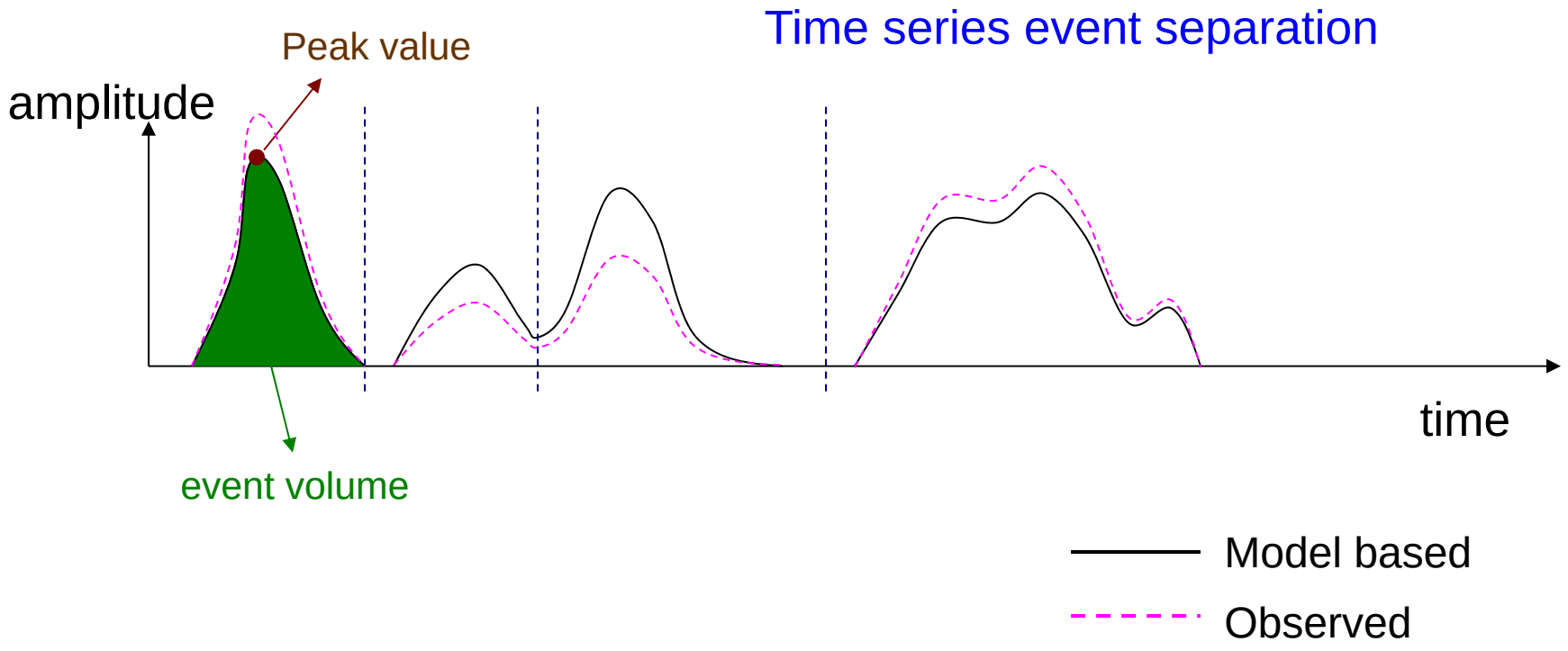
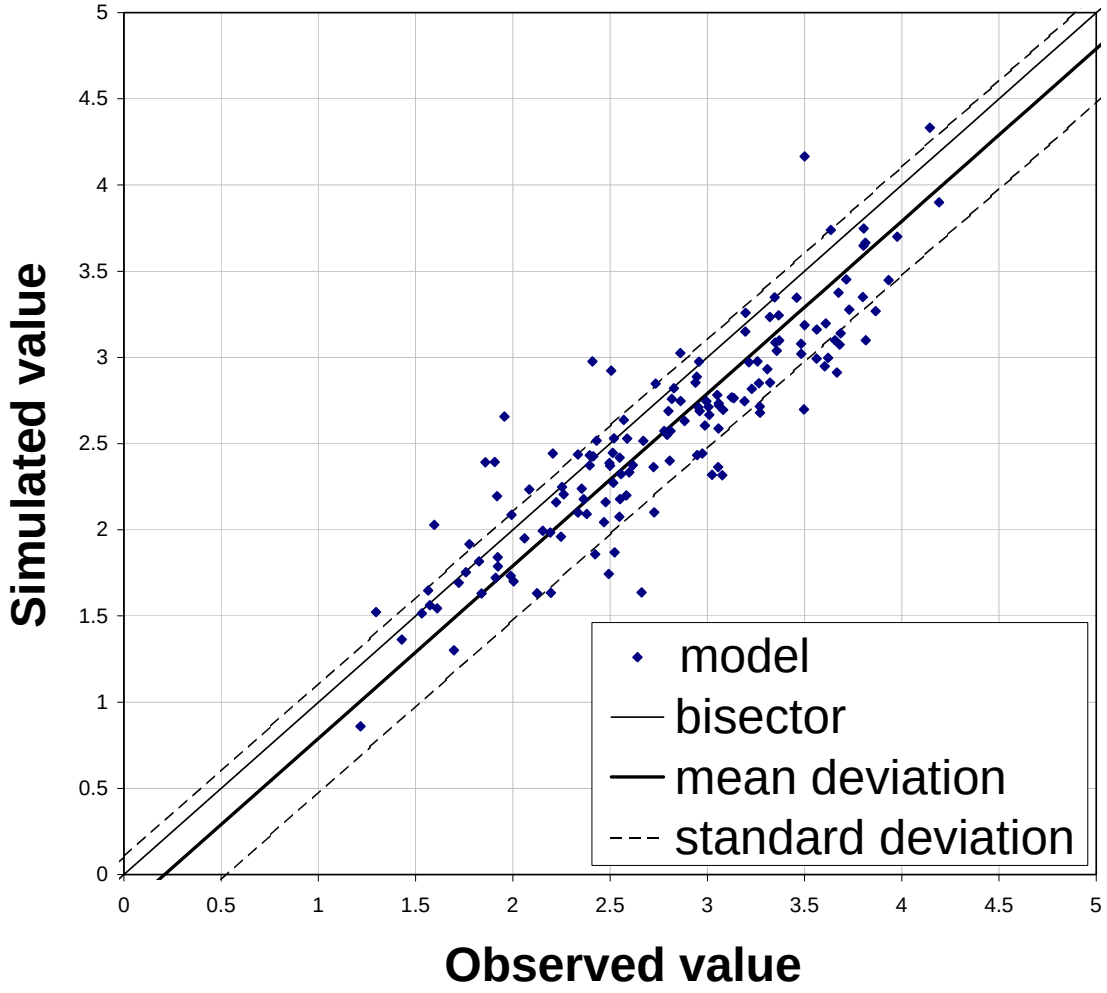


Willems P. (2008), 'Quantification and relative comparison of different types of uncertainties in sewer water quality modelling', **Water Research, 42**, 3539-3551

Total uncertainty quantification



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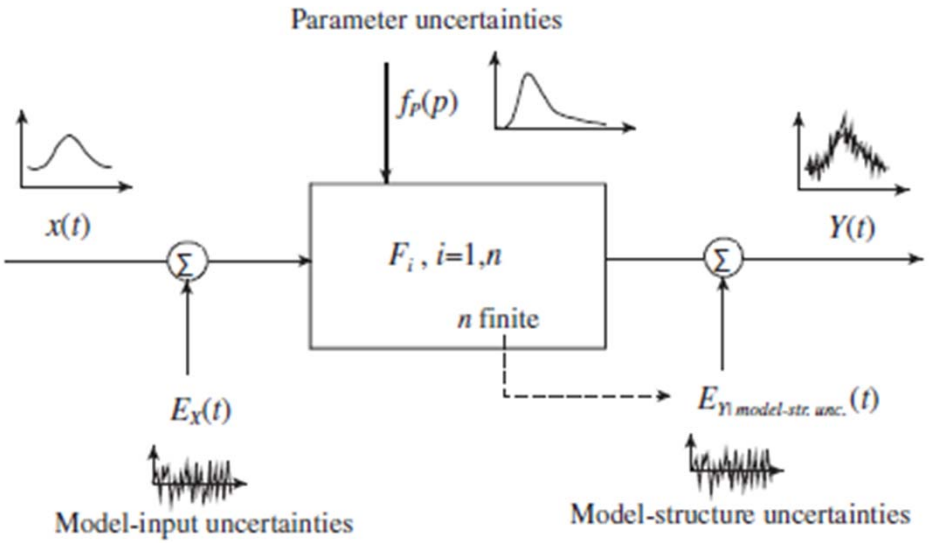
Model uncertainty analysis

Variance decomposition:

$$S^2_{E_{Y-Y_o}} = S^2_{E_{Y_o}} + S^2_{E_{Y|inputs\ X}} + S^2_{E_{Y|model\text{-}str.unc.}} + S^2_{E_{Y|parameters\ P}} + \ldots$$



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Model uncertainty analysis

Variance decomposition:

Model output: **Y**

Errors in inputs, parameters, model-structure: $\mathbf{X_1}$, $\mathbf{X_2}$, …, $\mathbf{X_k}$

$$V(Y) = \sum_i V_i + \sum_i \sum_{j>i} V_{ij} + \cdots + V_{12\ldots k}$$

Normalizing by V: first and higher order sensitivity indices:

$$\sum_i S_i + \sum_i \sum_{j>i} S_{ij} + \cdots + S_{12\ldots k} = 1$$

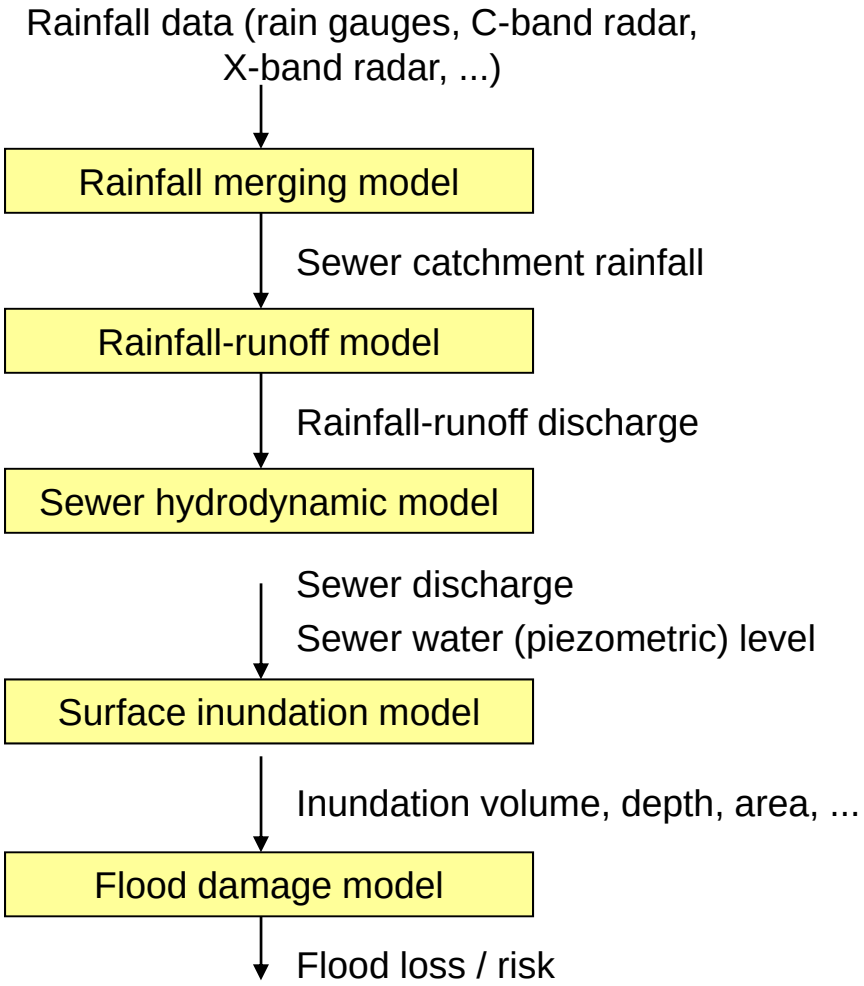
Total sensitivity index for X_i:

$$S_i + \sum_{j>i} S_{ij} + \sum_{l>j>i} S_{ijl} + \cdots + S_{12\ldots k} = S_{T_i}$$

Willems, P. (2012), 'Model uncertainty analysis by variance decomposition', **Physics and Chemistry of the Earth, 42-44**, 21-30

Model uncertainty analysis

Steps in sewer inundation modelling:

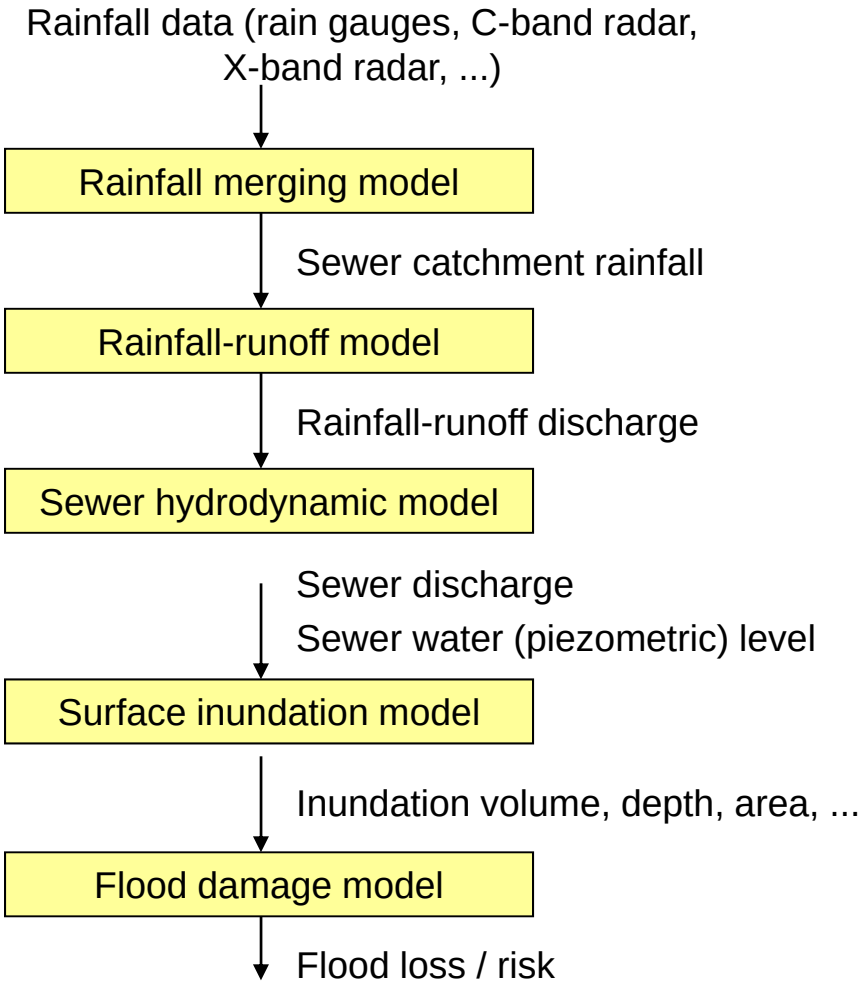


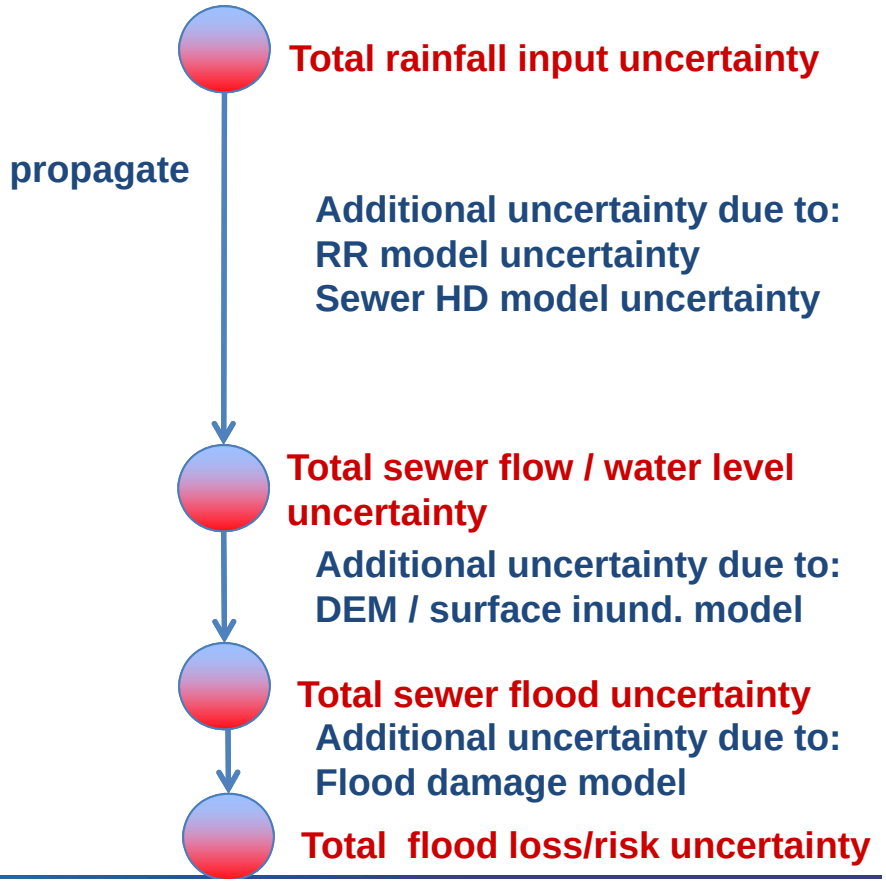
Rainfall input uncertainty

propagate

Model uncertainty analysis

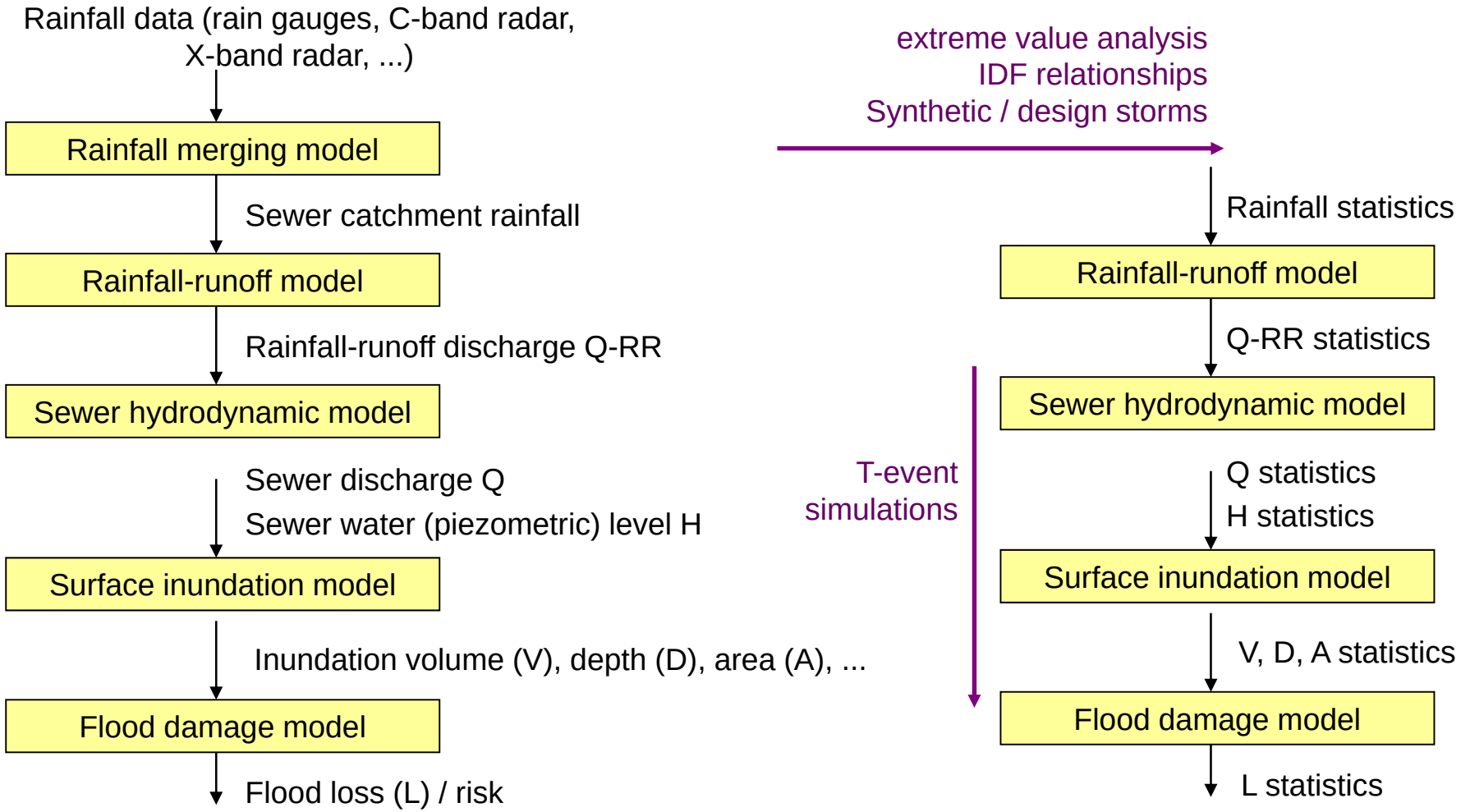
Steps in sewer inundation modelling:





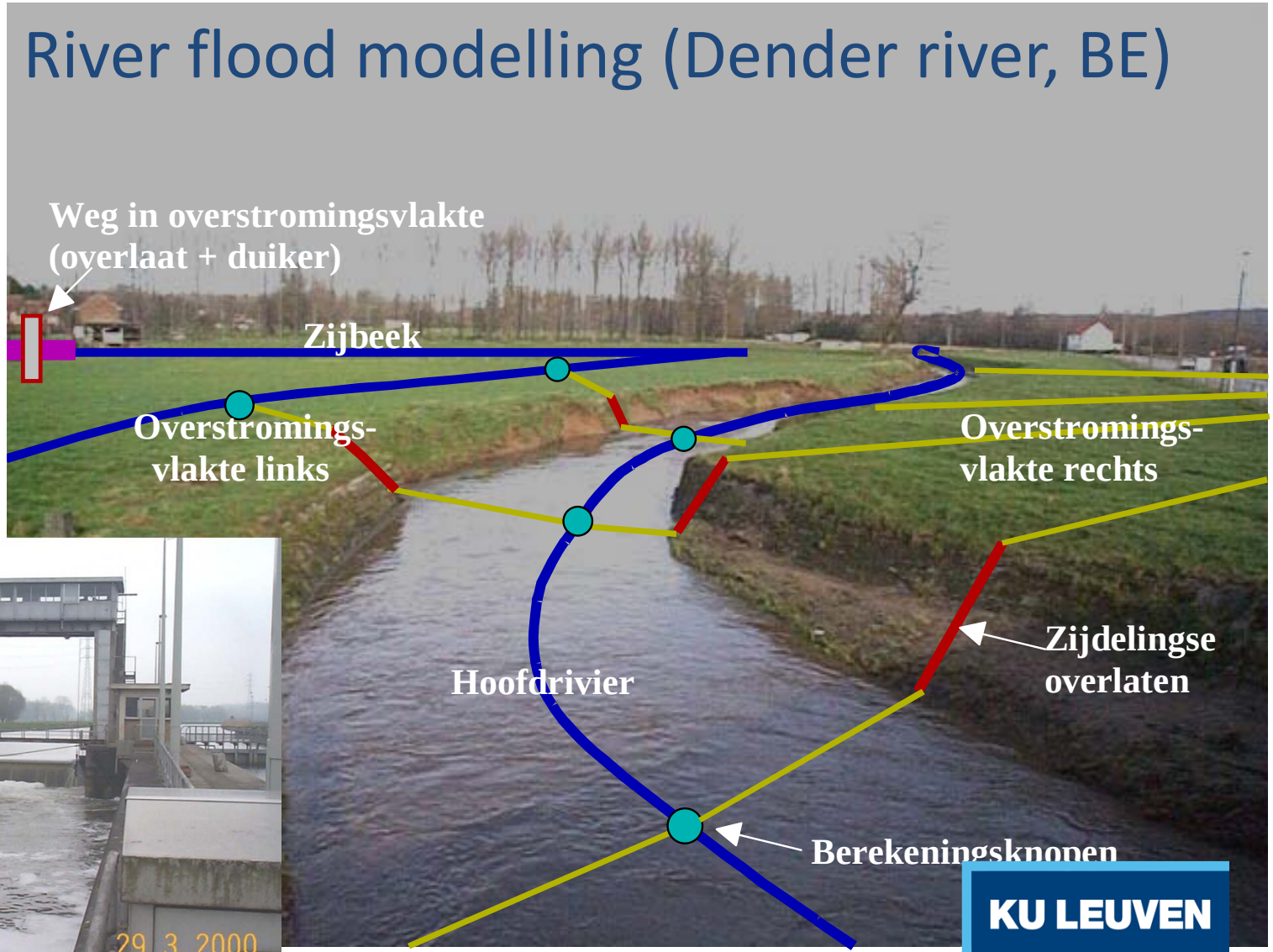
Model uncertainty analysis

Steps in sewer inundation modelling:



Model uncertainty analysis

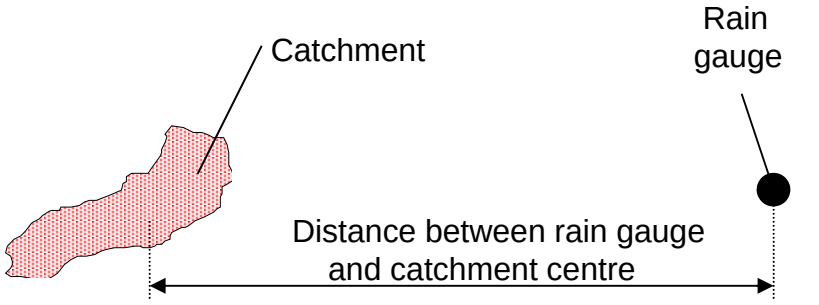
Example: River flood modelling (Dender river, BE)

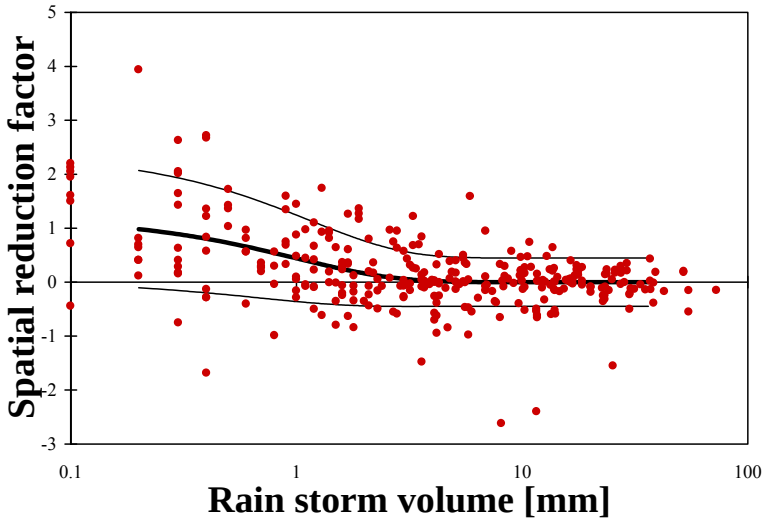


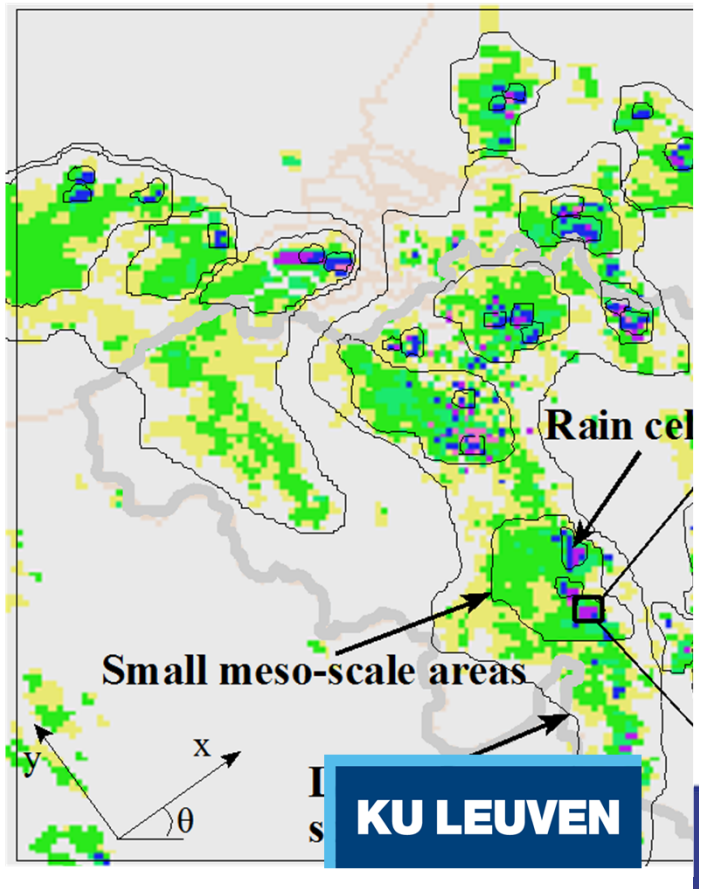
Model uncertainty analysis

Example: River flood modelling (Dender river, BE)

Total rainfall input uncertainty quantification:



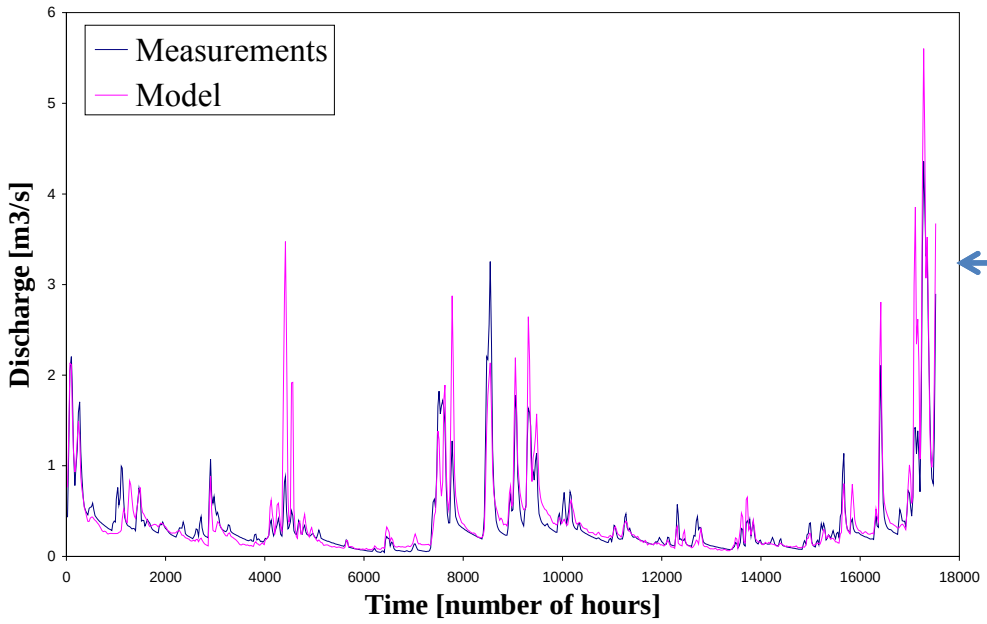


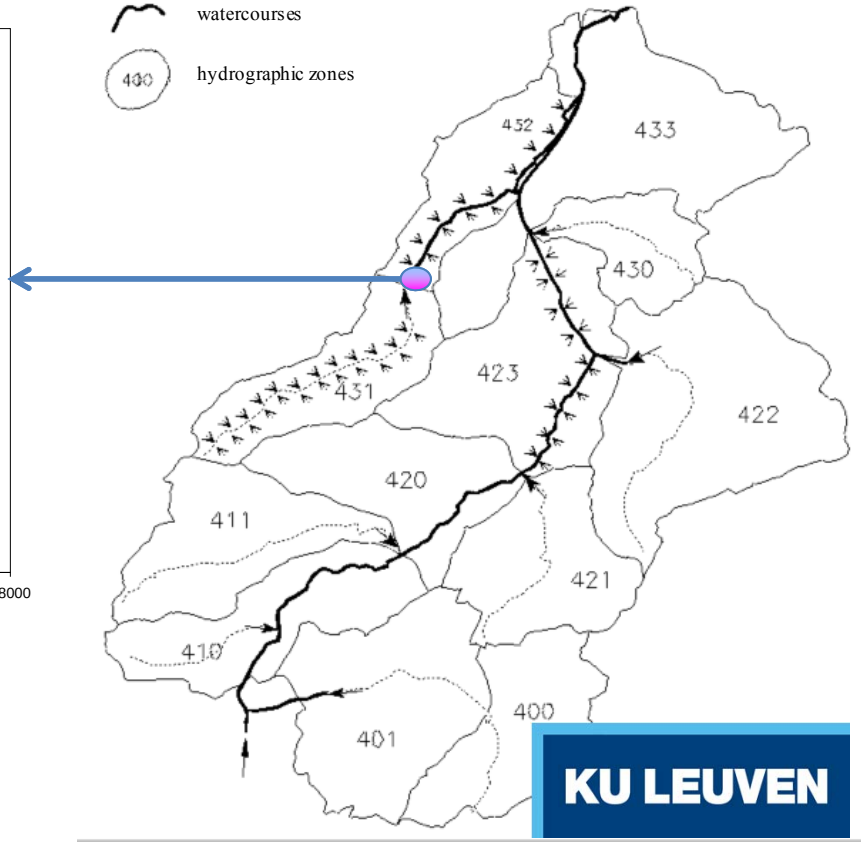


Model uncertainty analysis

Example: River flood modelling (Dender river, BE)

Total river flow uncertainty quantification:





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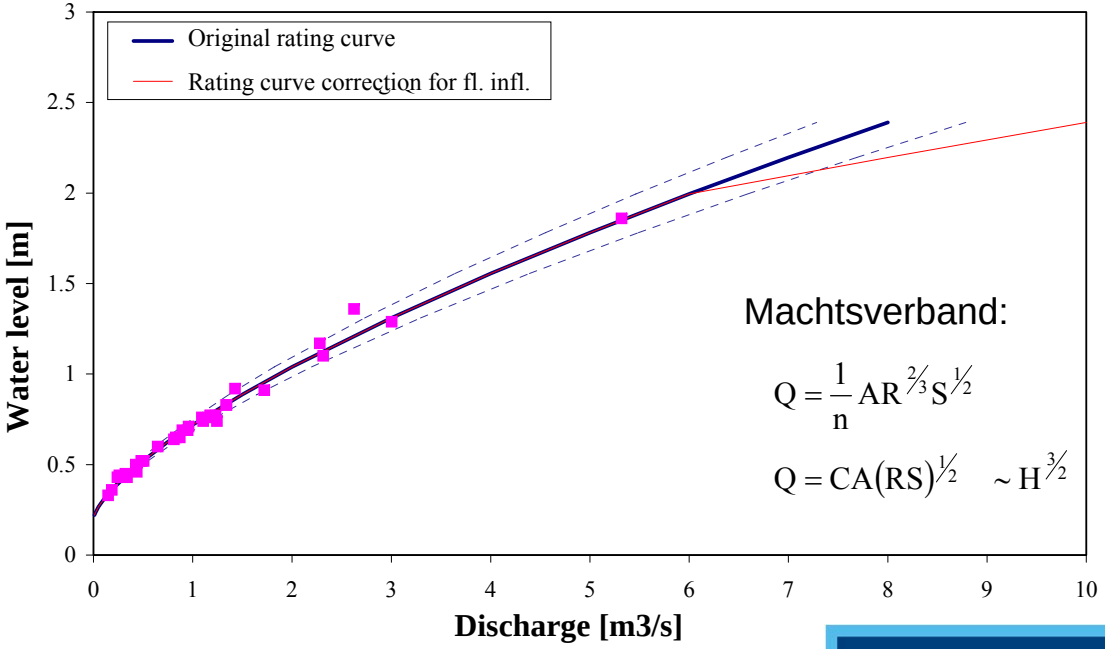
Model uncertainty analysis

Example: River flood modelling (Dender river, BE)

Take observation error variance into account:

Q-H rating curve uncertainty:



Machtsverband:

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

$$Q = CA(RS)^{1/2} \sim H^{3/2}$$

Model uncertainty analysis

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Example: River flood modelling (Dender river, BE)

Total flood uncertainty quantification:

$$F = \frac{A_{obs} \cap A_{\mathrm{mod}}}{A_{obs} \cup A_{\mathrm{mod}}}$$

Model uncertainty analysis

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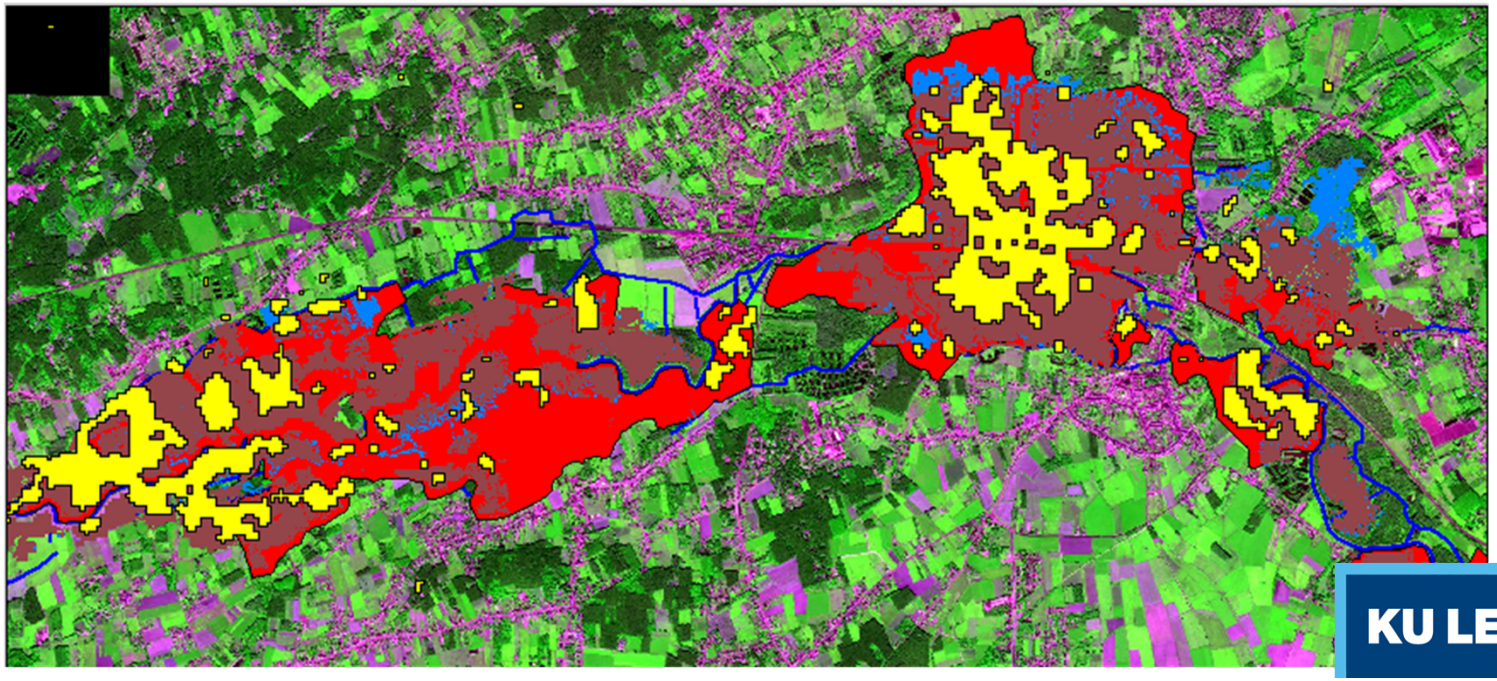
Example: River flood modelling (Dender river, BE)

Flood model validation

overlay of:

- *ERS SAR derived flood map at 30.01.1995*
- *Flemish Map of Recent Floods for the flood of January 1995 (at the peak moment)*
- *hydraulic model flood simulation results (at 30.01.1995)*
- *Hydraulic model flood simulation results (at the peak moment)*



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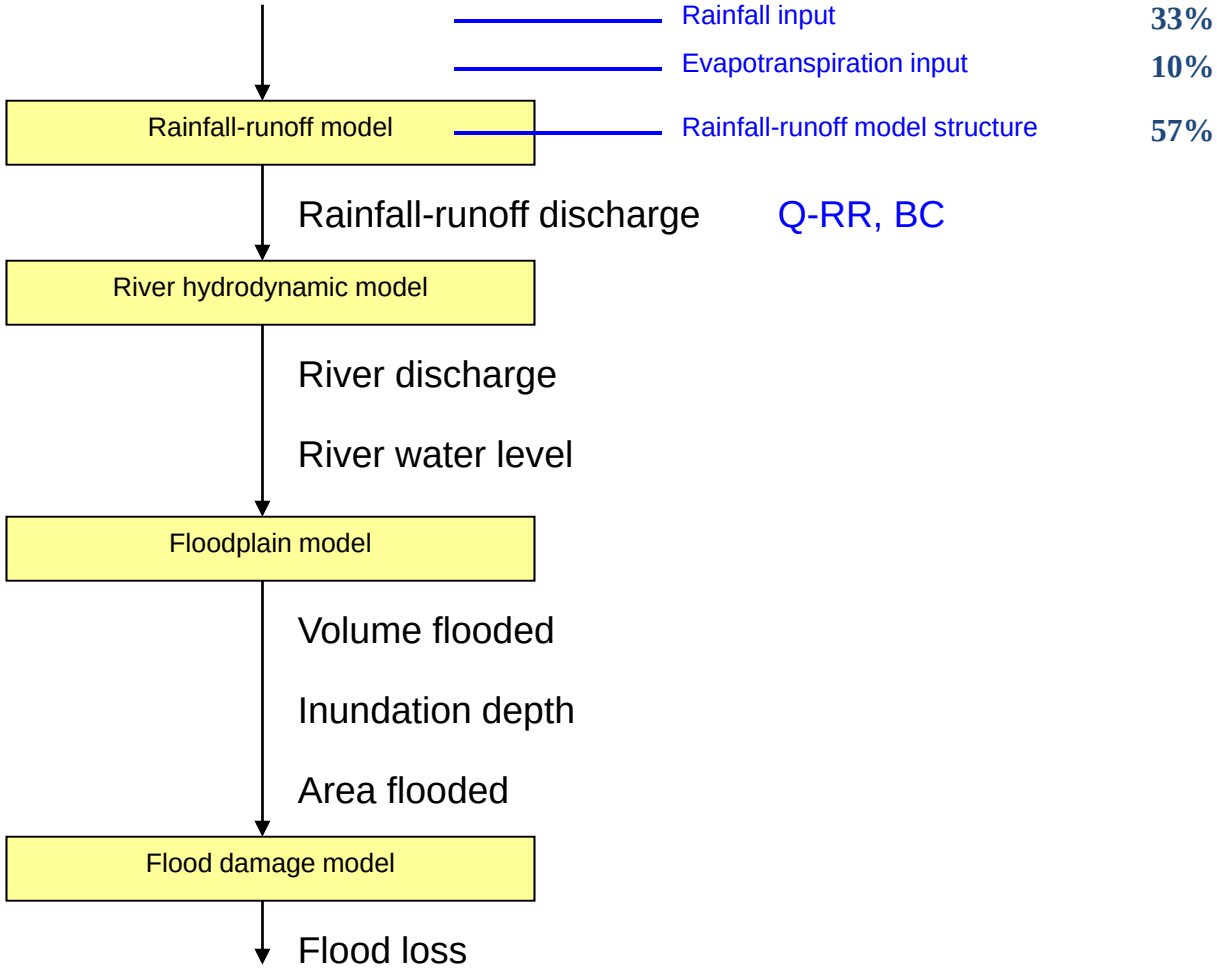

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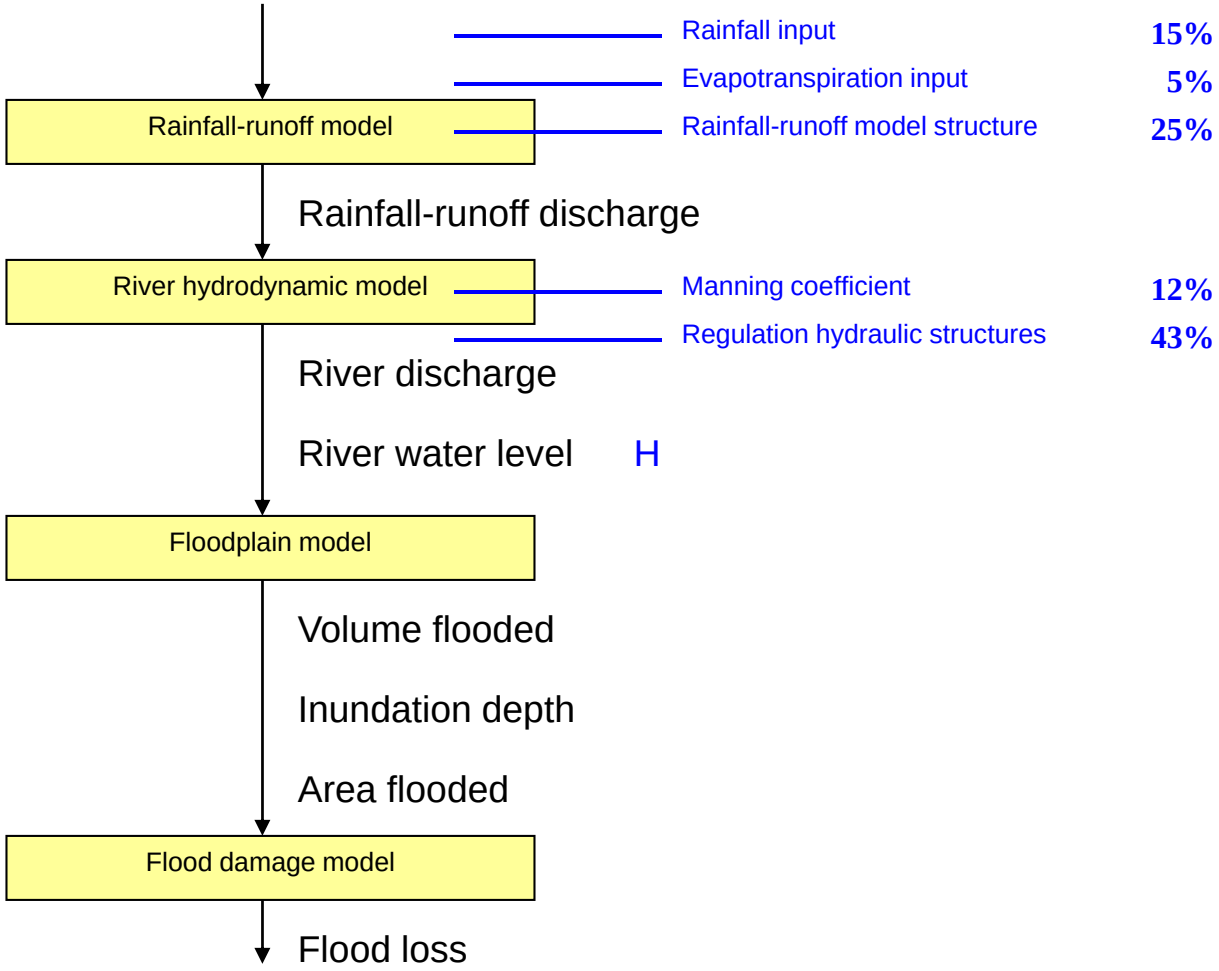

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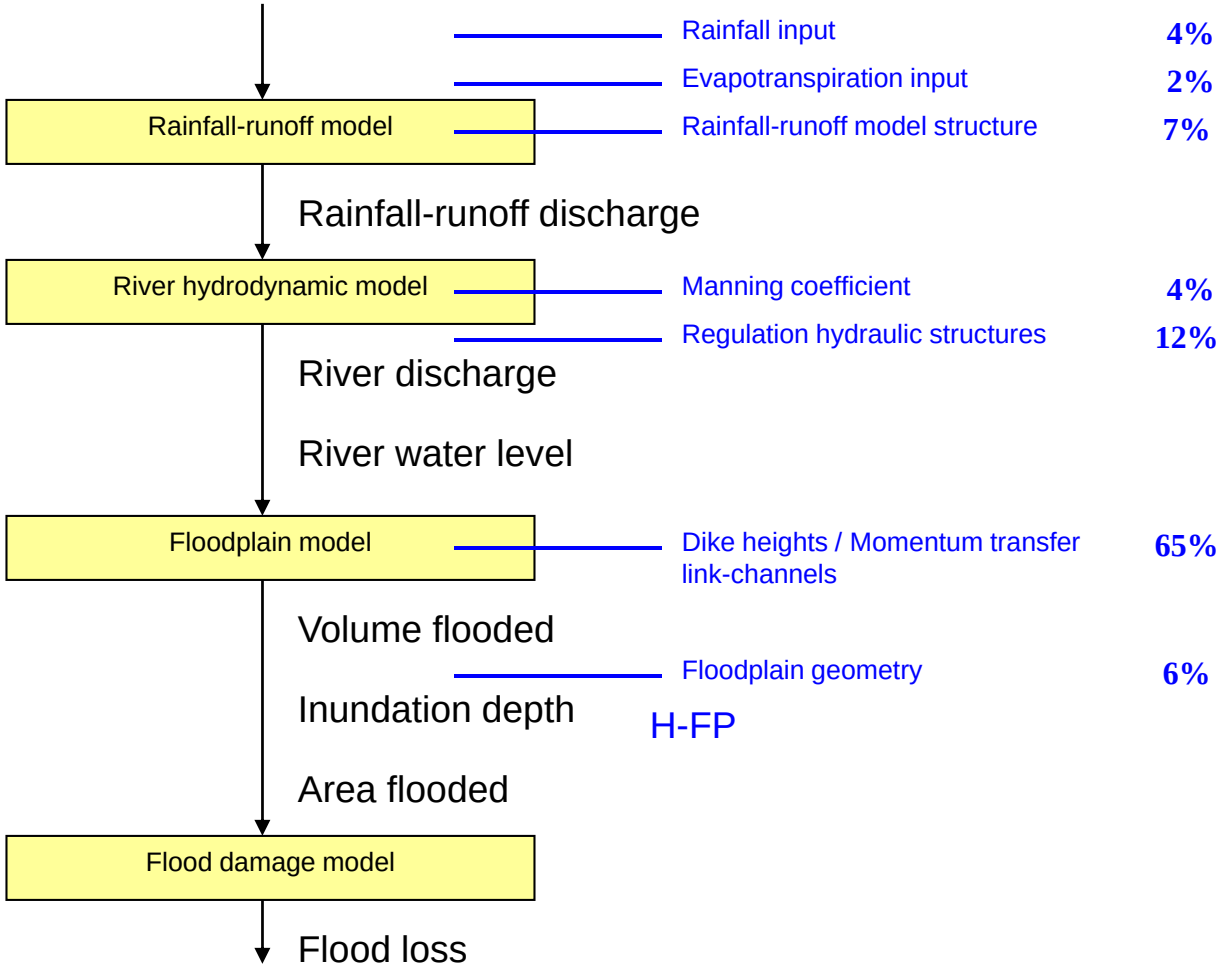
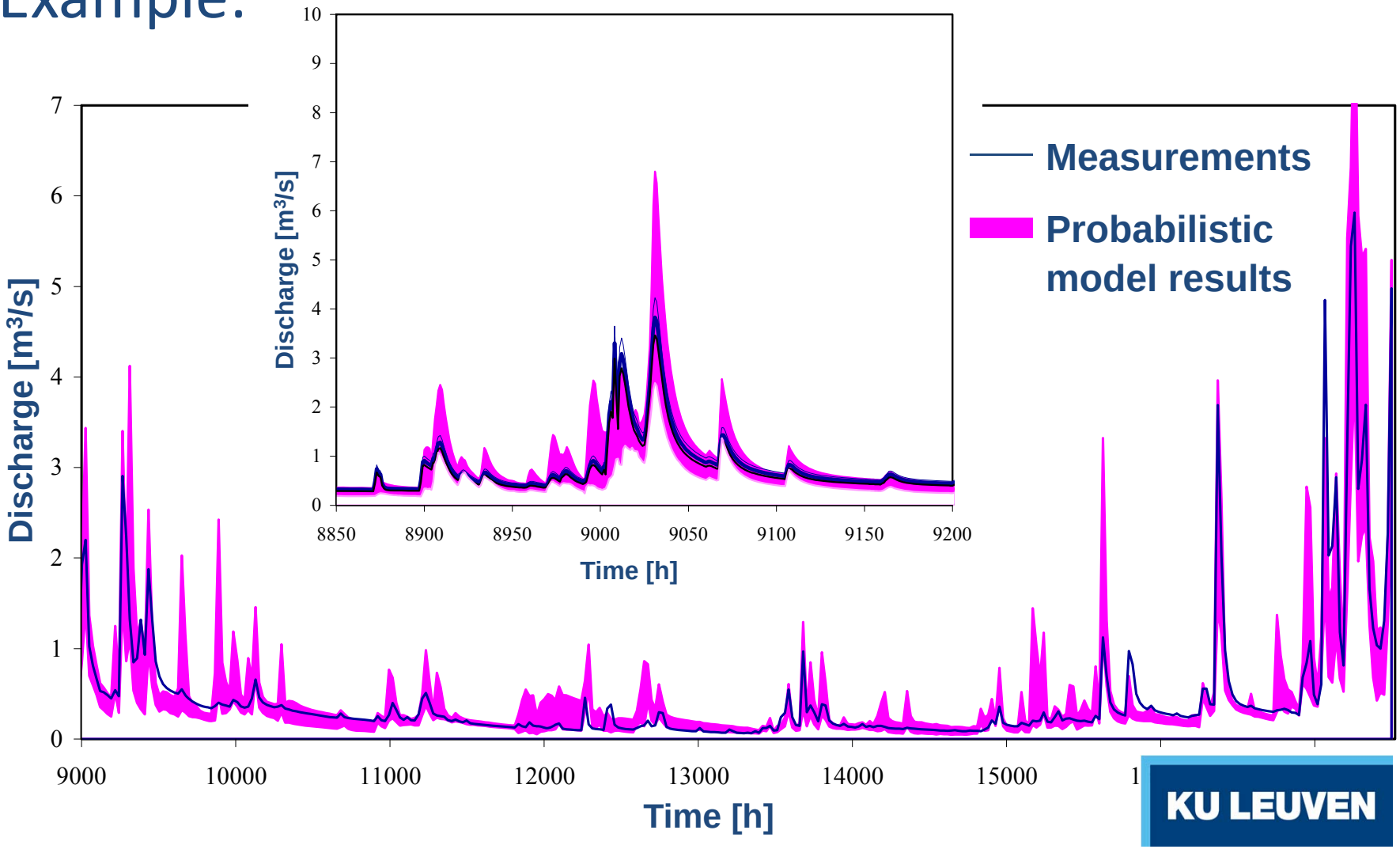


Model uncertainty analysis

Example: confidence limits on model results:



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Model uncertainty analysis

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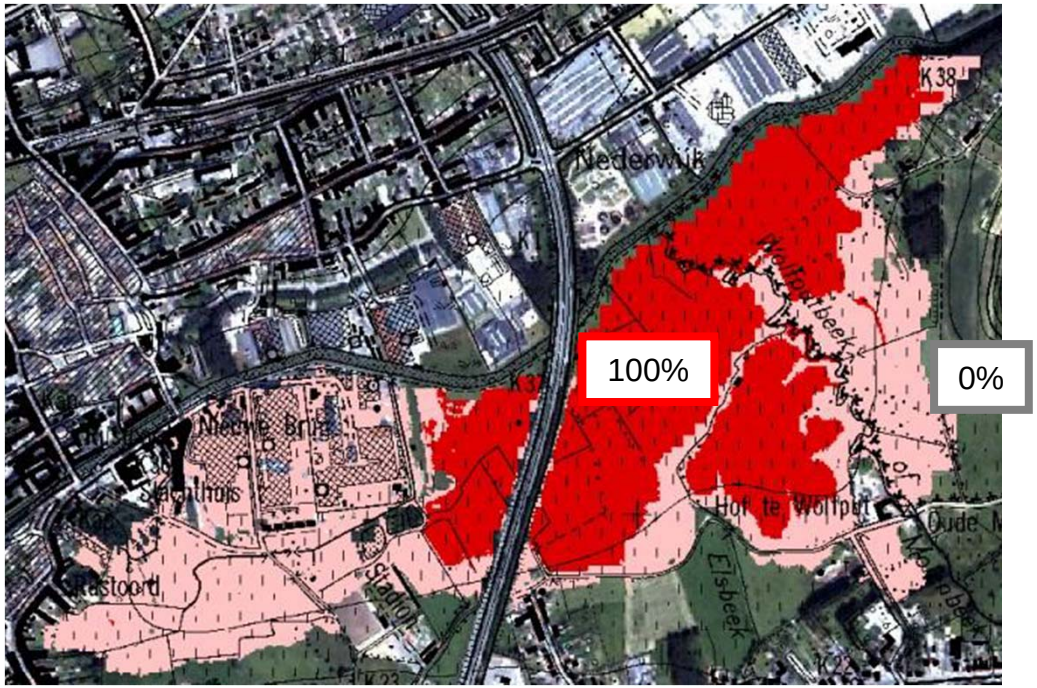
Example: river flood modelling (Dender river, BE)

Probabilistic flood maps:



Event-based flood map

0% -> 100% prob.

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Model uncertainty analysis

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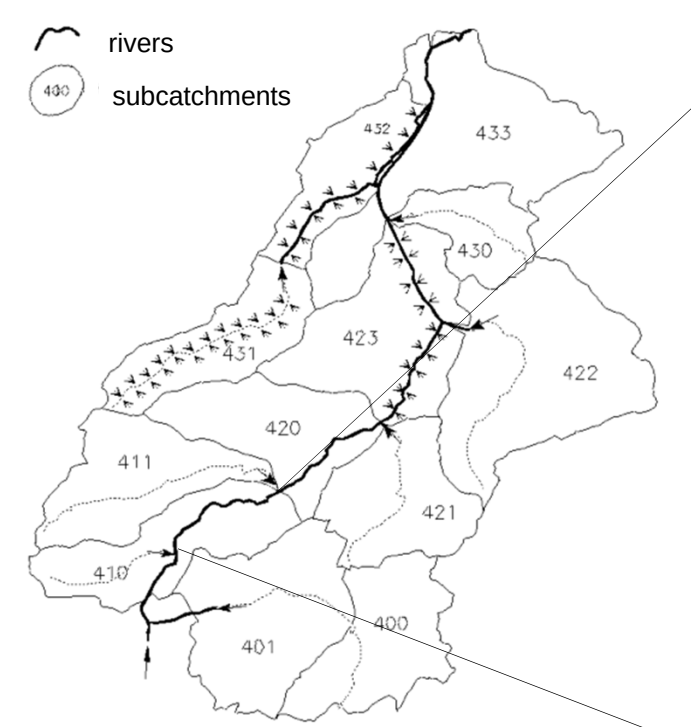
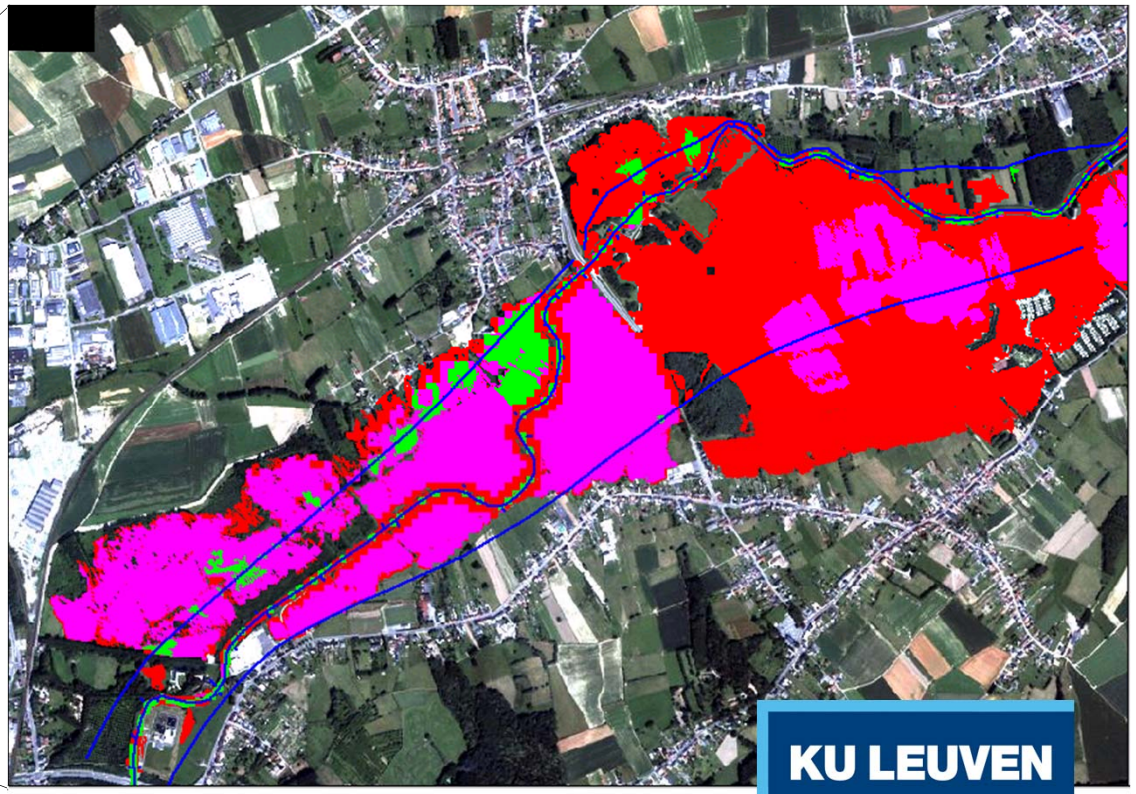
Example: river flood modelling (Dender river, BE)

Flood maps for: T=1year 10 years 100 years





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Rainfall forecast uncertainty quantification

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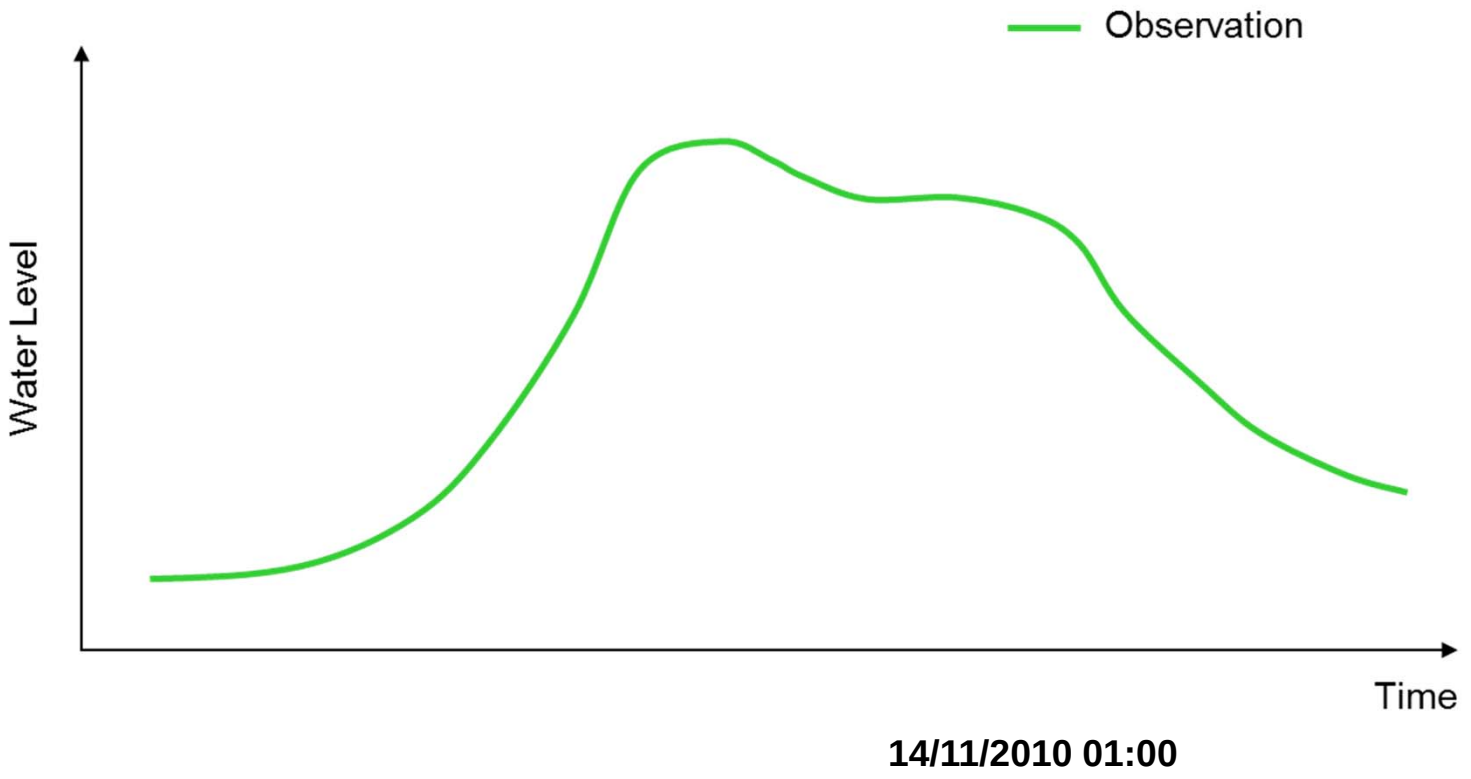
Statistical non-parametric data-based approach:



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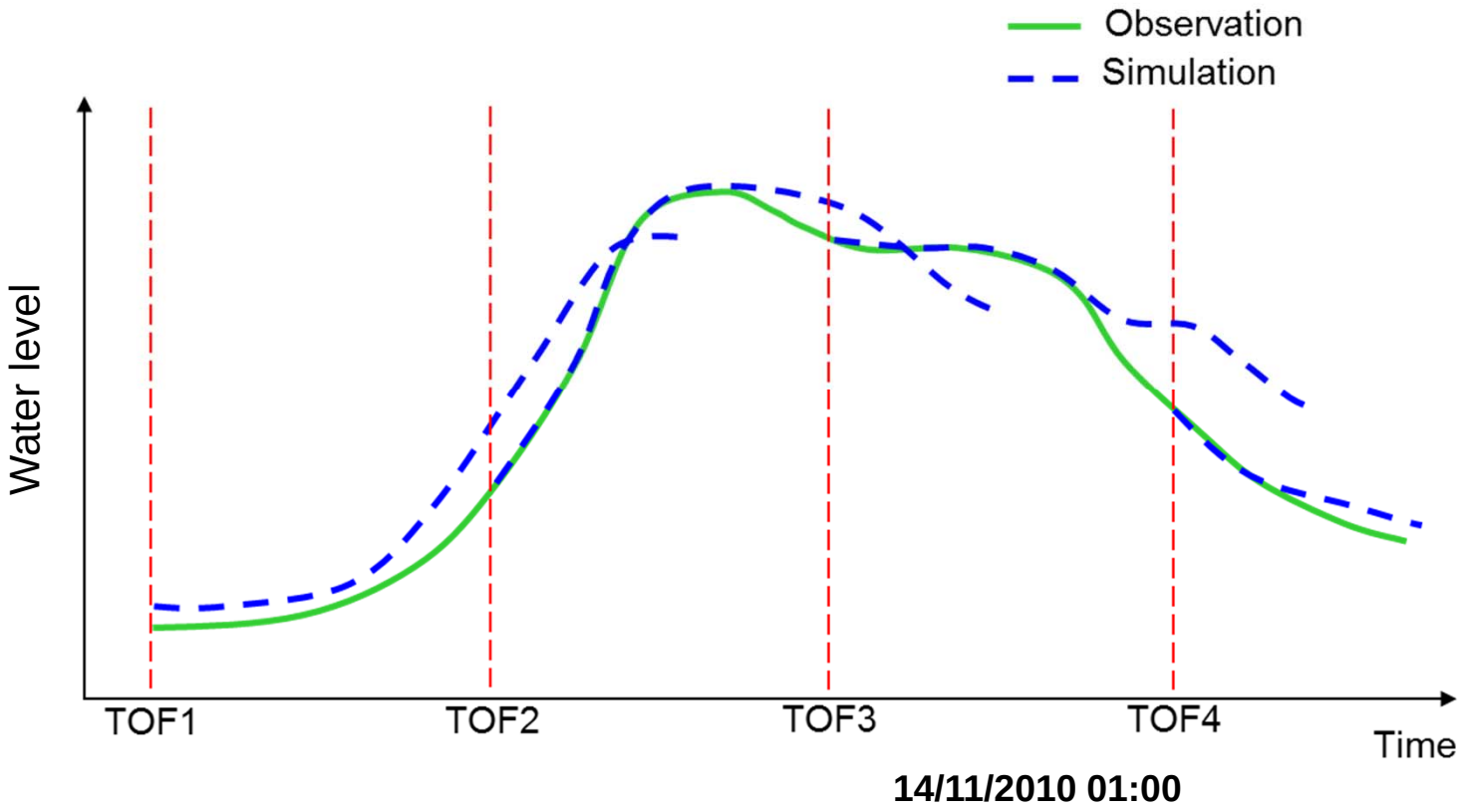

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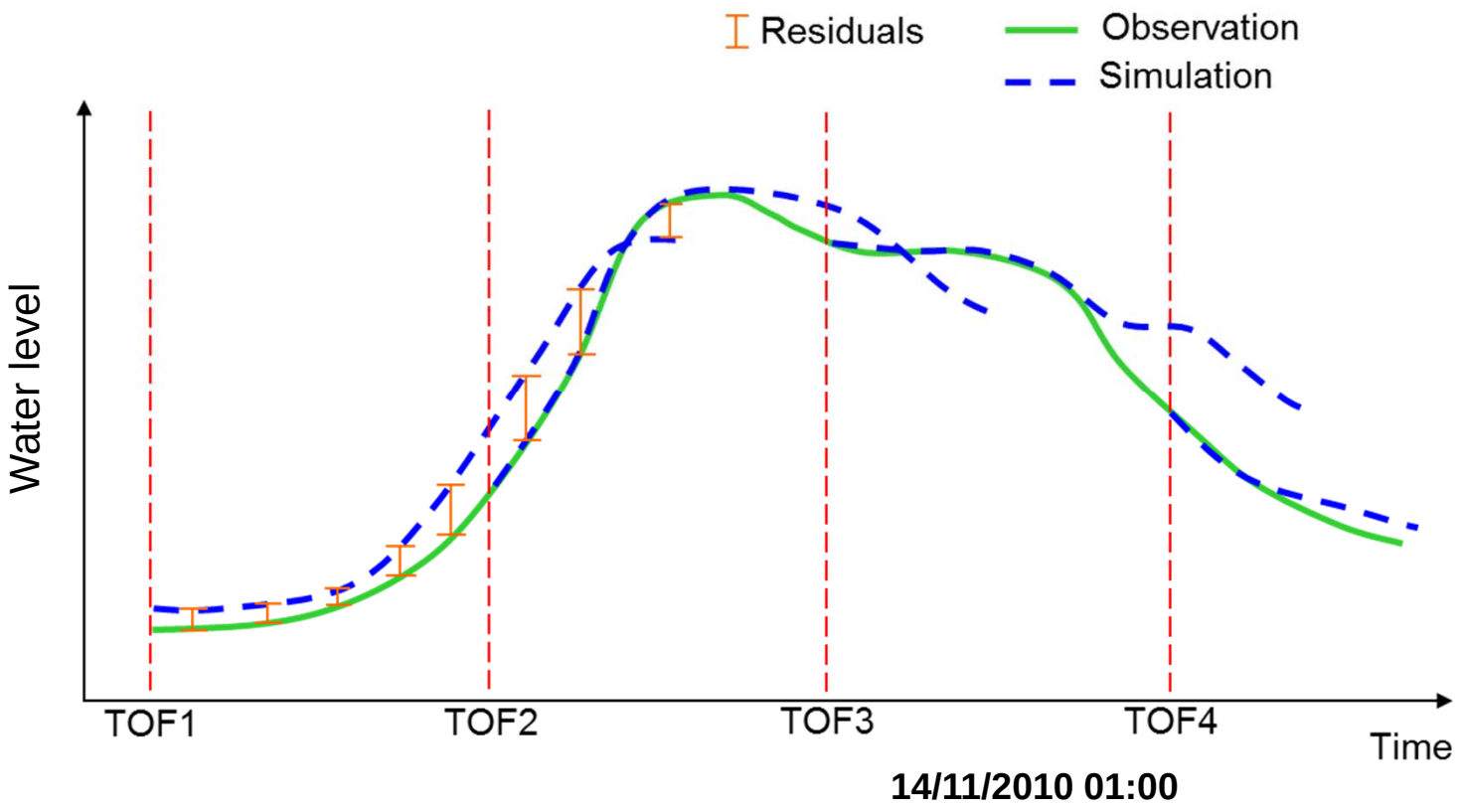

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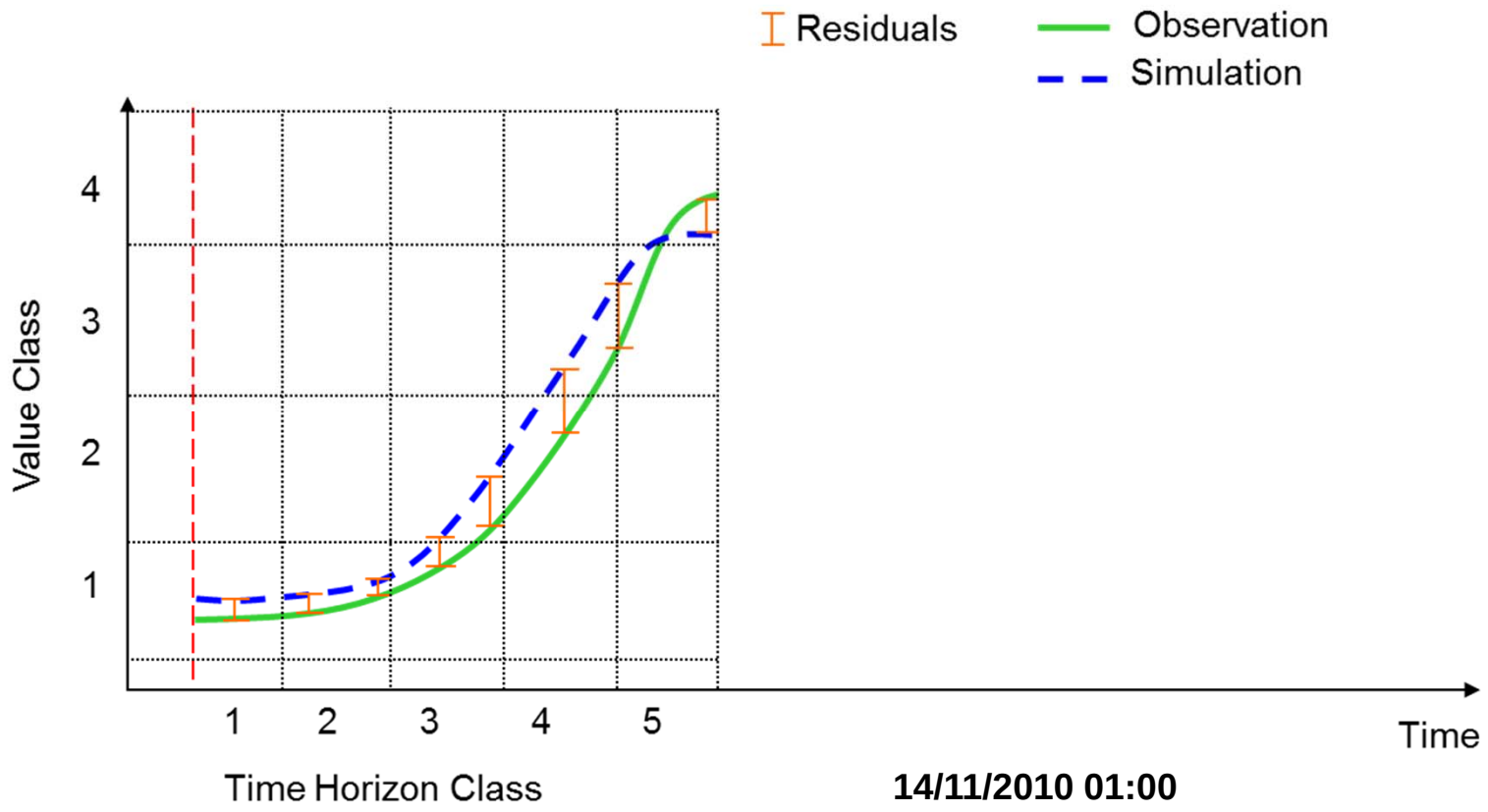

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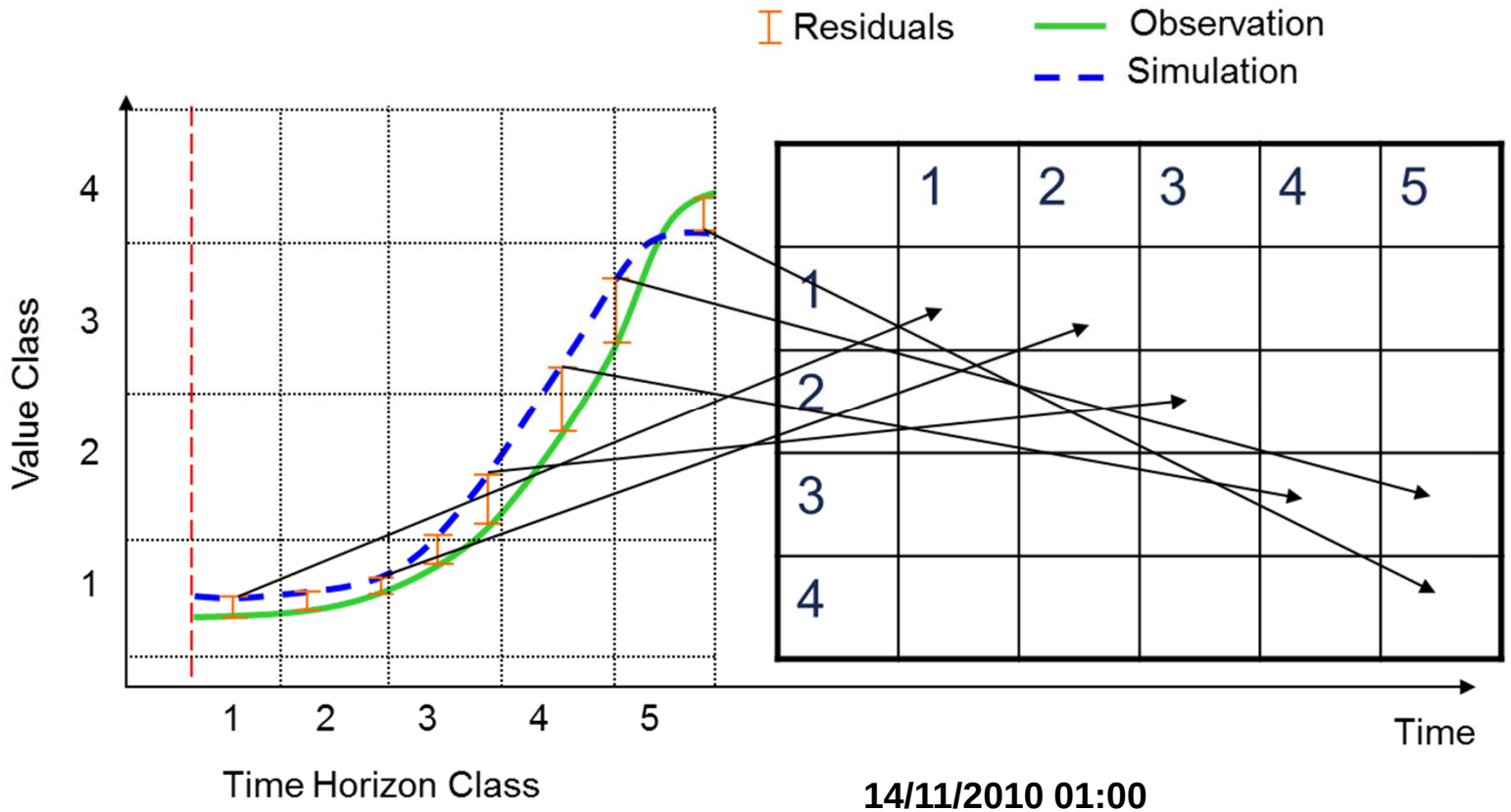

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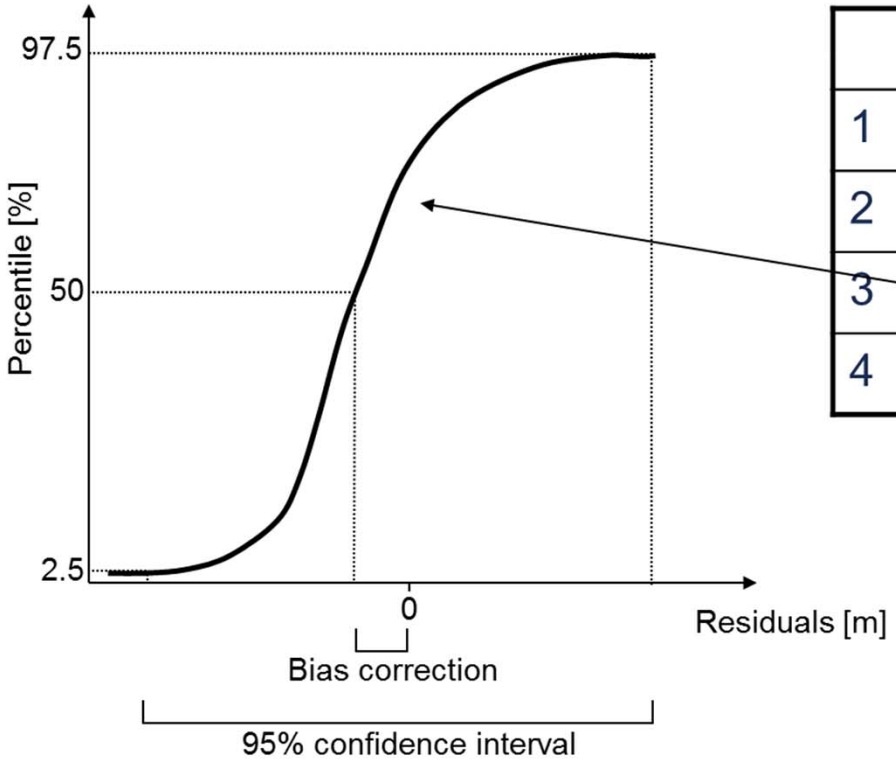

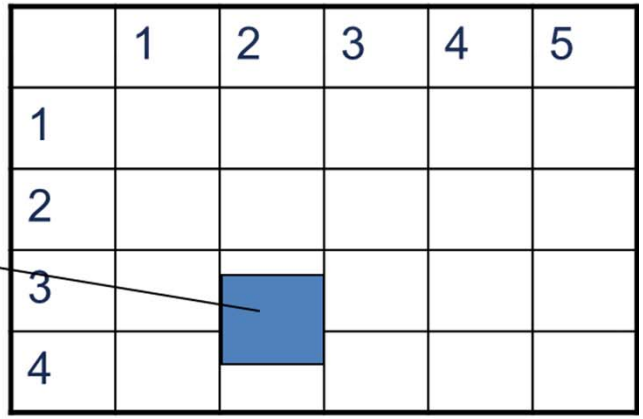

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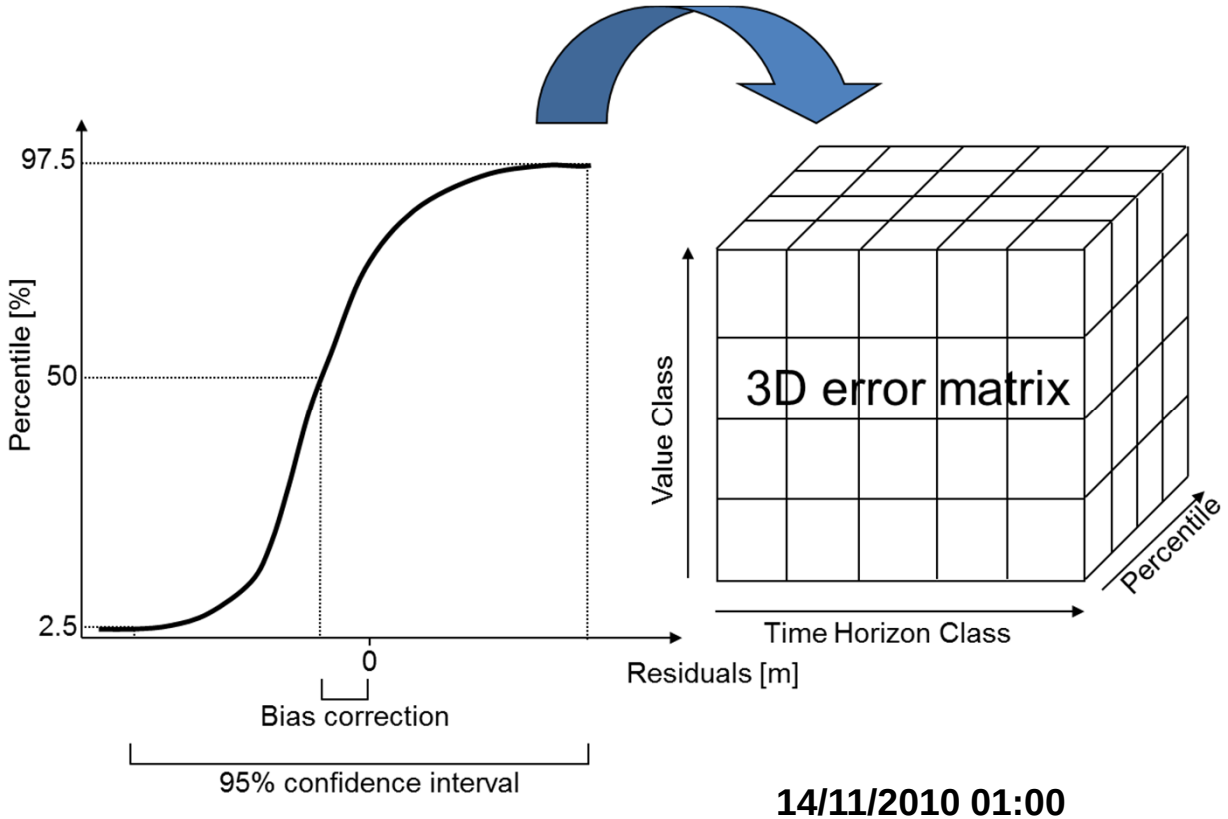

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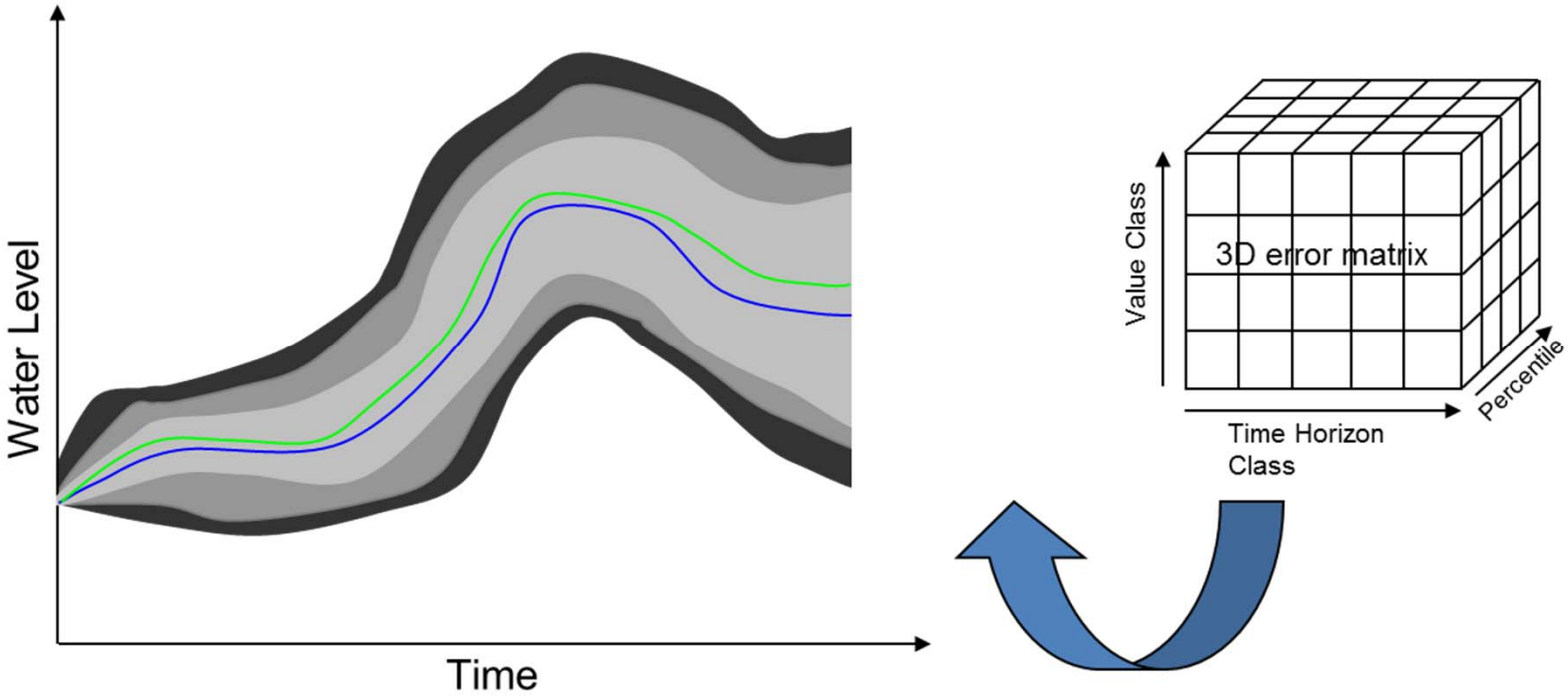

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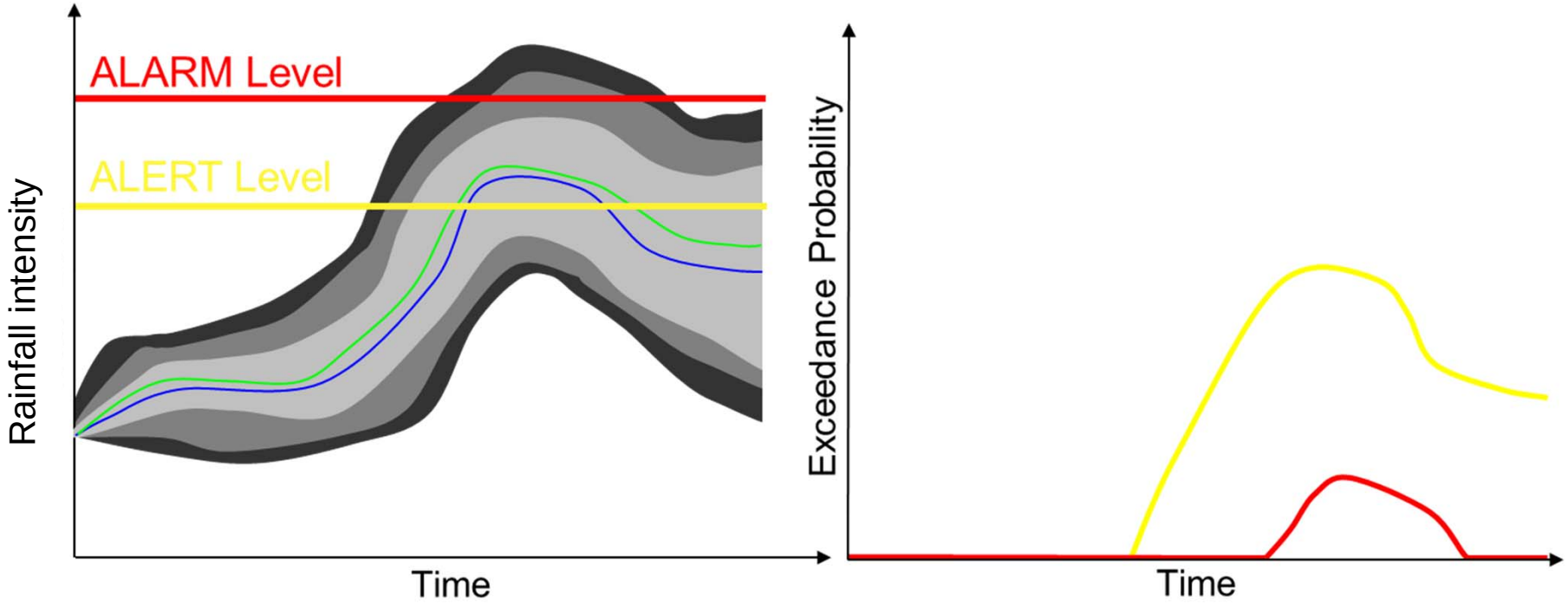

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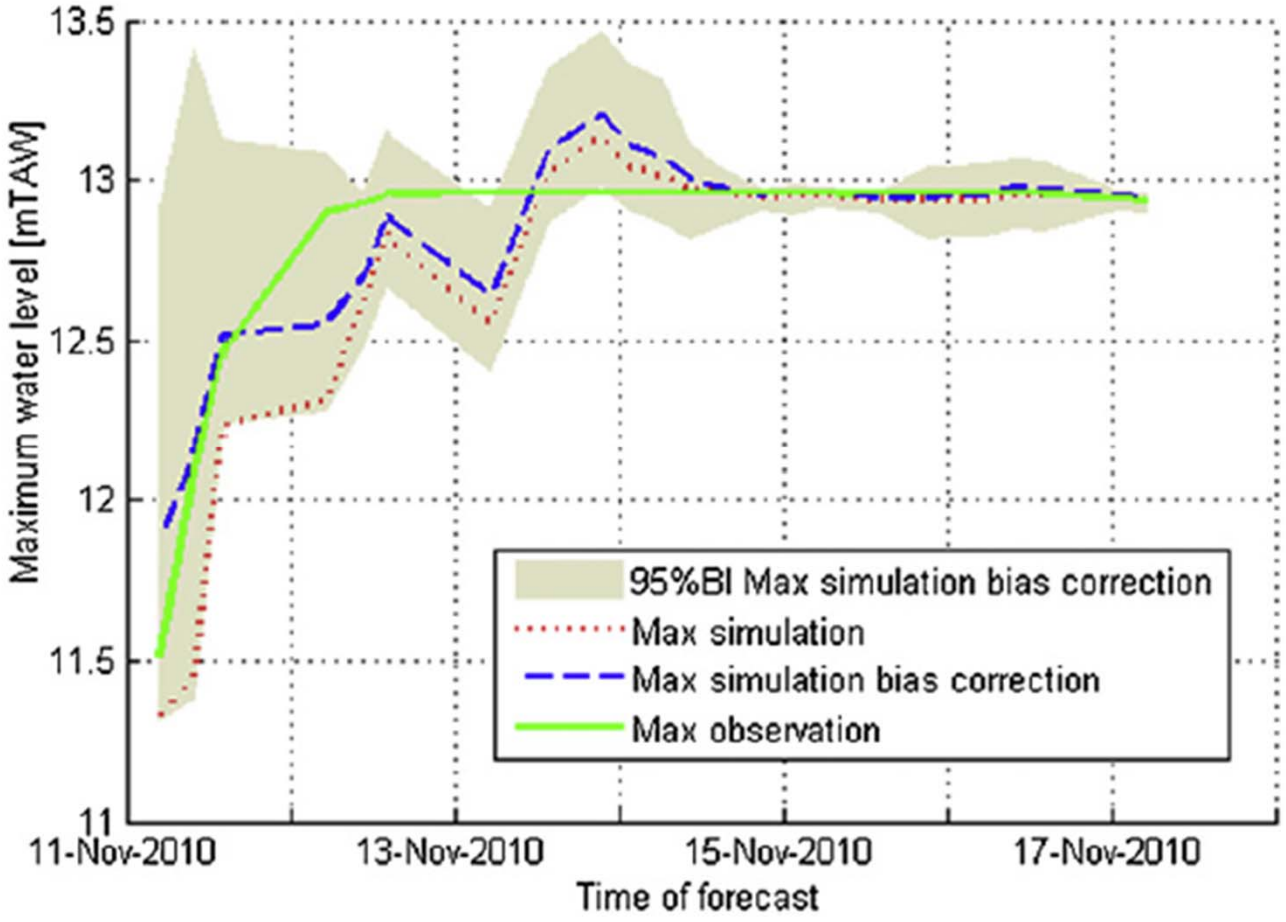

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Uncertainty visualisation

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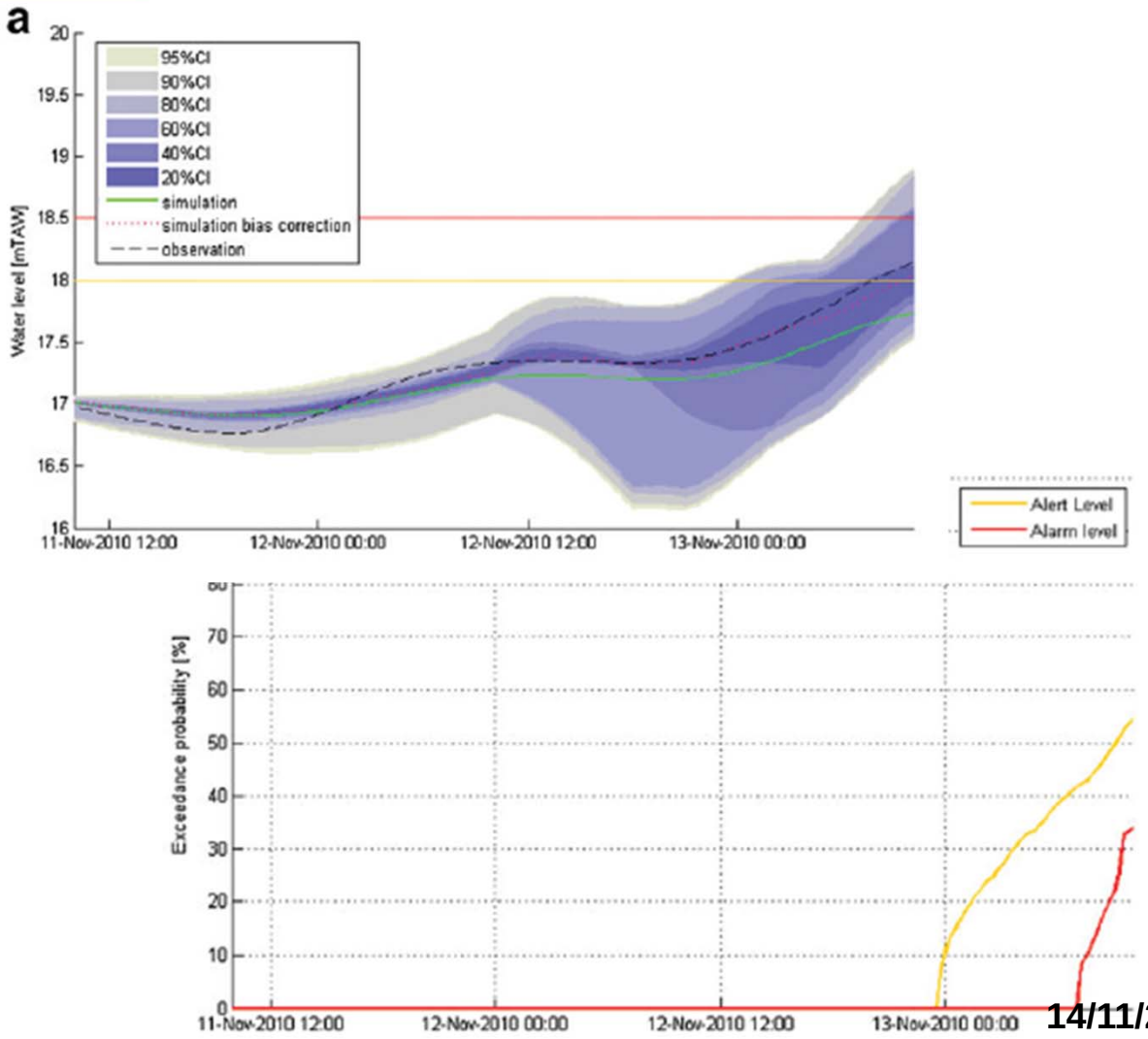
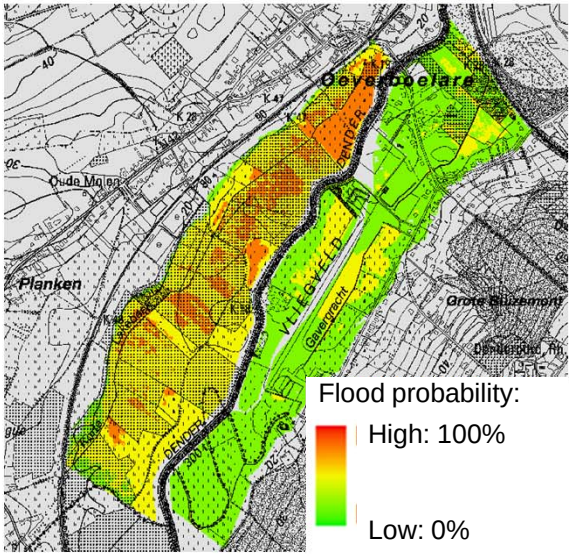


Flood probability map:



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Inundation uncertainty mapping

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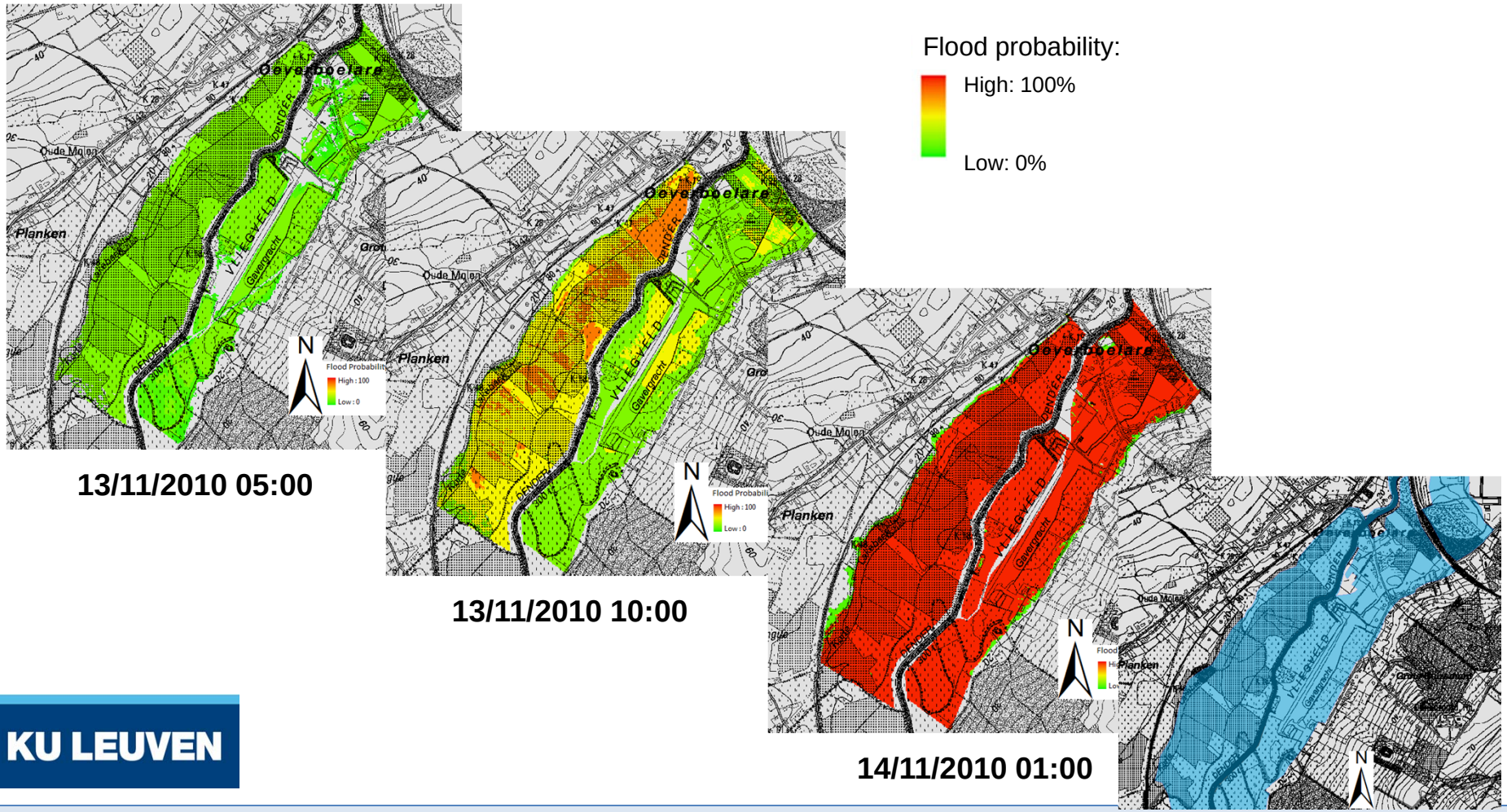
Flood probability map:



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