A toy model to deal with zero rainfall in a Universal Multifractal framework

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Chair "Hydrology for resilient cities" (sponsored by Veolia)

RainGain (EU INTER-REG NEW)

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Introduction : how to properly model the numerous zeros of rainfall fields

Rainfall variability modelled with the help of Universal Multifractals (UM) (see Lovejoy and Schertzer, 2007 for a review), which rely on the concept of multiplicative cascades

Three relevant scale invariant parameters:
- H: degree of non-conservation
- C_1: mean intermittency
- α : multifractality index

Rainfall intermittency: a succession of wet and dry periods

- 95 – 98% of zeros for high resolution (5min) long (few years) time series

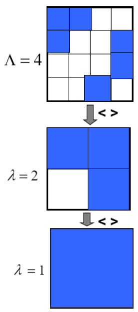
Disagreement in the literature on how to handle the zeros:

- How to model the zeros : multiplication by an independent support, a simple threshold, within the cascade process…
- A scaling break or not.
- Agreement that it biases the estimates of UM parameters (underestimation of α and overestimation of C_1), but not on how to retrieve the correct ones

Universal multifractal framework

Conservative fields

Fractal dimension D_F : $\text{number of pixels with rain} = \lambda^{D_F}$



$\Lambda = 4$: $< R_4^q >=$; $\Pr(R_4 \geq 4^\gamma)=$

$\lambda = 2$: $< R_2^q >=$; $\Pr(R_2 \geq 2^\gamma)=$

$\lambda = 1$: $< R_1^q >=$; $\Pr(R_1 \geq 1^\gamma)=$

< >

If multifractal:

$$< R_\lambda^{\,q} > \approx \lambda^{K(q)}$$
$$\Pr(R_\lambda \geq \lambda^\gamma) \approx \lambda^{-c(\gamma)}$$

Scaling moment function

Codimension function

$$resolution = \lambda = \frac{L}{l} = \frac{outer\_scale}{observation\_scale}$$

$$K(q) = \begin{cases} \frac{C_1}{\alpha - 1}(q^\alpha - q) & \alpha \neq 1 \\ C_1 q \ln q & \alpha = 1 \end{cases}$$

Estimation of UM parameters : the double trace moment technique (DTM) (Lavallée et al. ,1993)

$$\left\langle \left(\frac{R_\lambda^{\,\eta}}{\langle R_\lambda^{(\eta)} \rangle} \right)^q \right\rangle \approx \lambda^{K(q,\eta)} = \lambda^{\eta^\alpha K(q)}$$

Standard framework for non- conservative fields

Non-conservative → $\phi_\lambda = \varepsilon_\lambda \, \lambda^{-H}$ ⟹ $K(q) = K_c(q) - Hq$

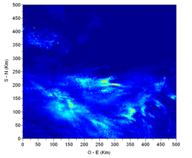
$\langle \phi_\lambda \rangle \approx \lambda^{-H}$

Conservative ($K_c(q)$)

Discrepancies with the theoretical framework

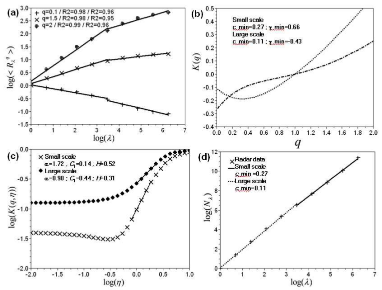
A scaling break (in D_F, and in the TM/DTM analysis)

Illustration with a heavy rainfall event (South of France, 16h, 5th Sept. 2005)



Total rainfall amount (mm)

Resolution : 1km x 1km x 15 min



H ~ 0.3 – 0.6 for many authors

→no physical explanation

Discrepancies between spatial and temporal analysis

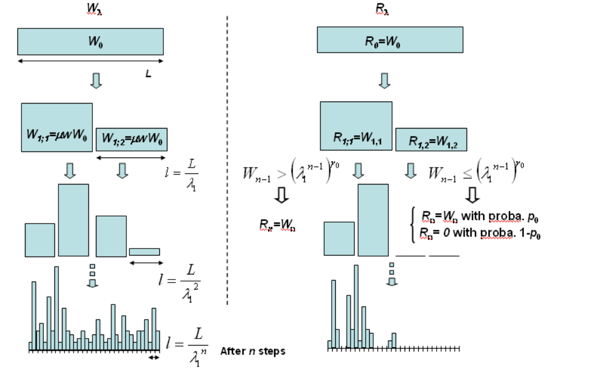
	Spatial (usually event based)	Temporal (usually long time series)
α	1.5 – 1.7	0.5 – 0.7
C_1	0.05 – 0.2	0.3 – 0.5
H	0.3 – 0.6	0 – 0.3

Effect of a simple threshold to reach 98.1% of zeros on simulated UM fields : quite disturbing !

UM parameters inputted	UM parameters retrieved
$\alpha = 1.7$; $C_1 = 0.2$	$\alpha = 0.55$; $C_1 = 0.38$
$\alpha = 0.6$; $C_1 = 0.45$	$\alpha = 0.45$; $C_1 = 0.44$

A new toy model : UM + 0 (+T)

Real zeros are generated within the cascade process :



$W_{n-1} > \left(\lambda_1^{\,n-1}\right)^{\gamma_0}$; $W_{n-1} \leq \left(\lambda_1^{\,n-1}\right)^{\gamma_0}$

$$\mu\varepsilon = \exp\left[\left(\frac{C_1 \ln(\lambda_1)}{|\alpha - 1|}\right)^{1/\alpha} L(\alpha)\right] / \lambda_1^{\frac{C_1}{\alpha - 1}}$$

- C_1 and α : wanted UM parameters
- $L(\alpha)$: extremal Lévy-stable random variable of index α, i.e. : $\langle \exp(qL(\alpha)) \rangle = \exp(q^\alpha)$

Underlying assumptions :

- Conserved quantity = W_λ (total water amount) and not the rainfall
- If W_λ too small, then R_λ not sure to survive

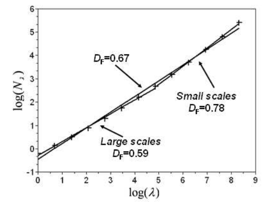
+ Threshold to mimic the limit of detection of rainfall measurement device

Results with C_1=0.1, α=1.9, γ_0=0.1 and p_0=0.5

- Ensemble analysis on 1000 independent samples of size 4096
- Threshold implemented on the normalized field

The rainfall support

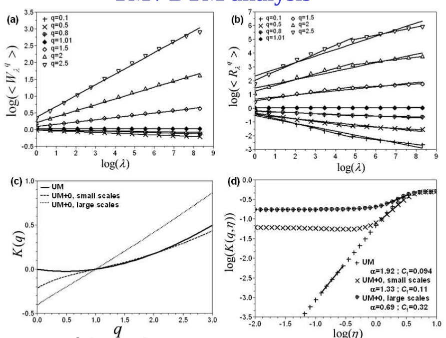
% of zeros : 95%, and 97% with T



D_F small scale > D_F large scale

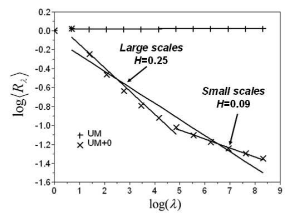
→ A limit of the model

TM / DTM analysis



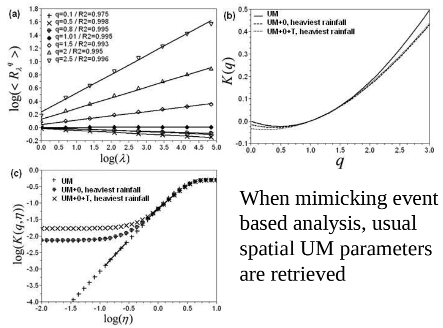
- Worsening of the scaling
- Behaviour small vs. large scales in agreement with observations
- Small scales more accurate to retrieve correct underlying UM field

Non-conservation H



- A model enabling to explain $H \neq 0$ for rainfall
- H small scales < H large scales (not observed)

Analysis on the heaviest rainfall



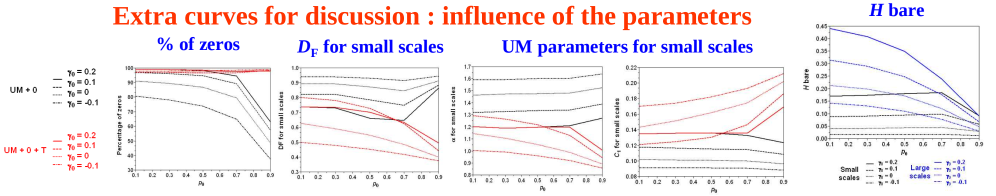
When mimicking event based analysis, usual spatial UM parameters are retrieved

Conclusion :

Although imperfect this simple toy model enables to reproduce / explain most of the discrepancies with the theoretical framework

Extra curves for discussion : influence of the parameters

% of zeros · D_F for small scales · UM parameters for small scales · H bare



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