QPE 233

# Non-Negative $K_{DP}$ Estimation by Monotone Increasing $\Phi_{DP}$ Assumption below Melting Layer



**Takeshi Maesaka*, Koyuru Iwanami and Masayuki Maki**

*National Research Institute for Earth Science and Disaster Prevention (NIED), Japan*


## INTRODUCTION

A specific differential phase ($K_{DP}$) is a powerful dual-polarimetric parameter for QPE. The $K_{DP}$ is not directly observed by radar system, but is calculated by a derivative of a differential phase ($\Phi_{DP}$) with respect to range. Because the observed $\Phi_{DP}$ is contaminated by noise, and the differentiation works as a high-pass filter, it is difficult to retrieve the original $K_{DP}$. Thus local linear or polynomial regression are generally used to estimate the $K_{DP}$; however, these procedures make a spatial resolution of the $K_{DP}$ coarse. Furthermore, a differential backscatter phase ($\delta$) overlaps in the observed $\Phi_{DP}$.

In this paper, a new $K_{DP}$ estimation method is proposed by assuming the monotone increasing $\Phi_{DP}$ below melting layer. The $K_{DP}$ estimated by this method always takes a positive value.

## METHOD

### 1. Quality Control

At first, the observed data in or higher than melting layer should be rejected, because this method is only available for pure rainfall. Of course, a precipitation particle identification can be used by using the dual-polarimetric information; however it is not completely robust at present. The zero degree level information from sounding data or numerical simulations, and the assumed melting layer depth (e.g., 500 m or 1000 m) may be useful for this purpose.

Then no precipitation (low S/N ratio) data, ground and point clutter data, and outlier data are rejected. Finally $\Phi_{DP}$ unfolding should be performed in case the $\Phi_{DP}$ exceeds a expression range of 360 °or 180 °.

### 2. Boundary Conditions

This method needs two boundary conditions: the nearest and farthest $\Phi_{DP}$ ($\Phi_{near}$ and $\Phi_{far}$). The final solution of $\Phi_{DP}$ varies between these boundary conditions.

The nearest boundary $\Phi_{near}$ is determined by the linear regression (Line LR–N in schematic profile above), which is calculated with the specified number (e.g., 30) of available data from the nearest range. If the slope of the regression line is positive, the value of the regression line at the nearest range ($r_{near}$) is used for the boundary condition. Otherwise, the averaged value of the available data is used. The farthest boundary $\Phi_{far}$ is determined in the same manner (see Line LR-F in the schematic profile).

### 3. Cost Function

Now the observed and final solution of differential phase are denoted as $\Psi_i$ and $(\Phi_{DP})_i$, respectively, where the suffix i is an index of range (i = 0, 1, 2, ... , N). Here we define $\phi_i$ as,

$$\phi_i = (\Phi_{DP})_i - \Phi_{near}. \quad (1)$$

This $\phi_i$ can be written with $K_{DP}$ as,

$$\phi_0 = 0, \quad (2)$$

$$\phi_i = 2\sum_{j=0}^{i-1}(K_{DP})_j \Delta r \qquad (i = 1, 2, 3, \cdots, N). \quad (3)$$

Because we assume that $K_{DP}$ always takes a positive value, we introduce $k_i$ as,

$$k_i^2 = 2(K_{DP})_i \Delta r. \quad (4)$$

So (3) can be written with $k_i$ as,

$$\phi_i = \sum_{j=0}^{i-1} k_j^2 \qquad (i = 1, 2, 3, \cdots, N). \quad (5)$$

On the other hand, the reverse version of $\phi_i$ is also defined as,

$$\phi'_i = \Phi_{far} - (\Phi_{DP})_i. \quad (6)$$

This $\phi'_i$ can be written with $k_i$ as,

$$\phi'_i = \sum_{j=i+1}^{N} k_j^2 \qquad (i = 0, 1, 2, \cdots, N-1), \quad (7)$$

$$\phi'_N = 0. \quad (8)$$

The differences between observation and the boundaries are also defined as,

$$\psi_i = \Psi_i - \Phi_{near}, \qquad \psi'_i = \Phi_{far} - \Psi_i. \quad (9 \text{ and } 10)$$

Hear a cost function (with respect to k) to be minimized is defined as,

$$J = J_{obs} + J'_{obs} + J_{lpf}, \quad (11)$$

$$J_{obs} = \frac{1}{N}\sum_{i=1}^{N}(\phi_i - \psi_i)^2,$$

$$J'_{obs} = \frac{1}{N}\sum_{i=0}^{N-1}(\phi'_i - \psi'_i)^2,$$

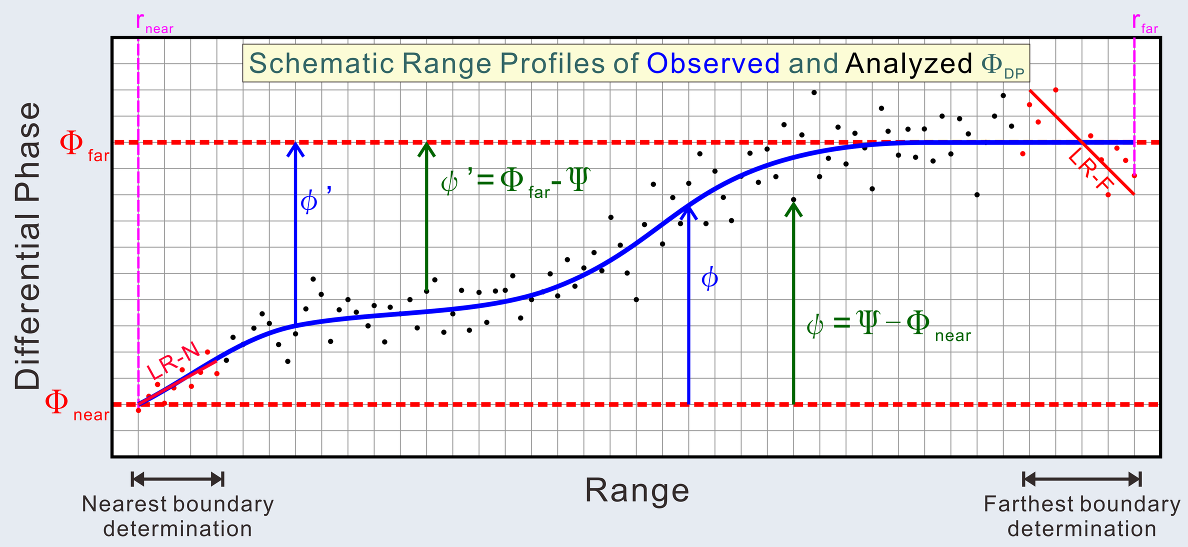
Mean Square Errors of the Observed $\Phi_{DP}$

$$J_{lpf} = \frac{1}{N+1}C_{lpf}\sum_{i=0}^{N}\left(\frac{\partial^2 k_i}{\partial r^2}\right)^2.$$

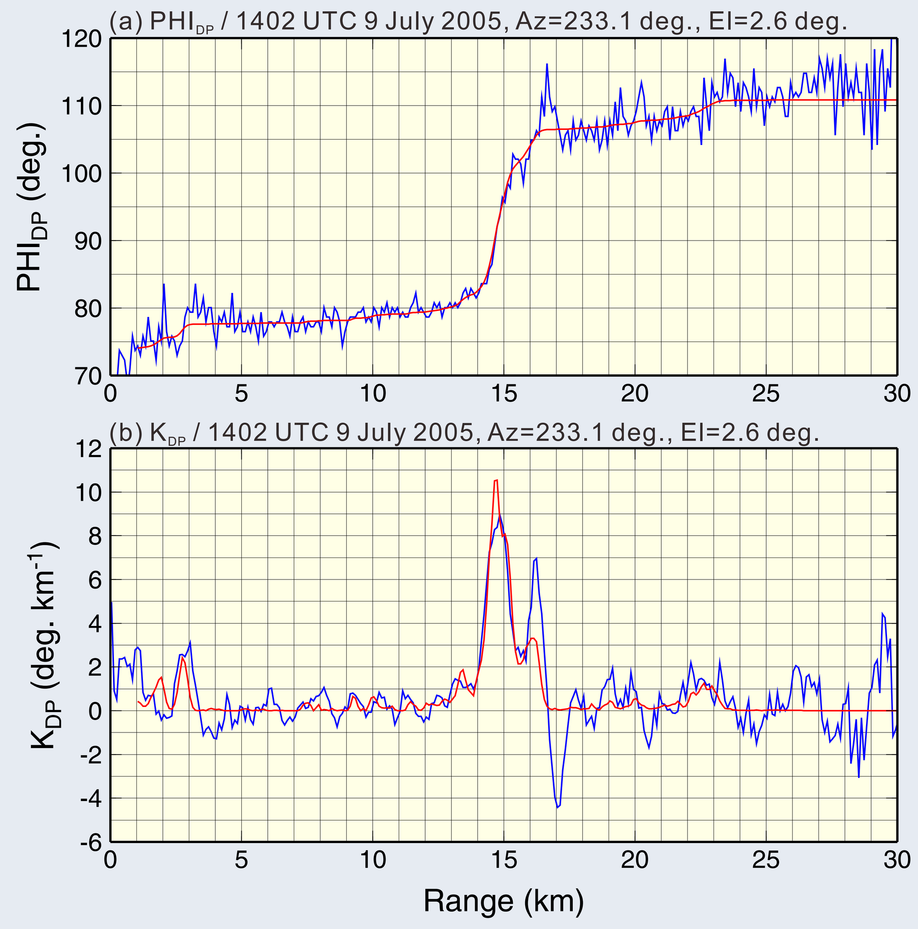
Mean Square of Laplacian of k, Worked as a Low-Pass Filter

$J_{obs}$ and $J'_{obs}$ make the analyzed differential phase fitted to the observed one. $J_{lpf}$ works as a low pass filter. $C_{lpf}$ is a control parameter of the low pass filter.

We regard the range profile of k which minimizes the cost function as the final solution in this problem. The final $K_{DP}$ is calculated from k by (4).

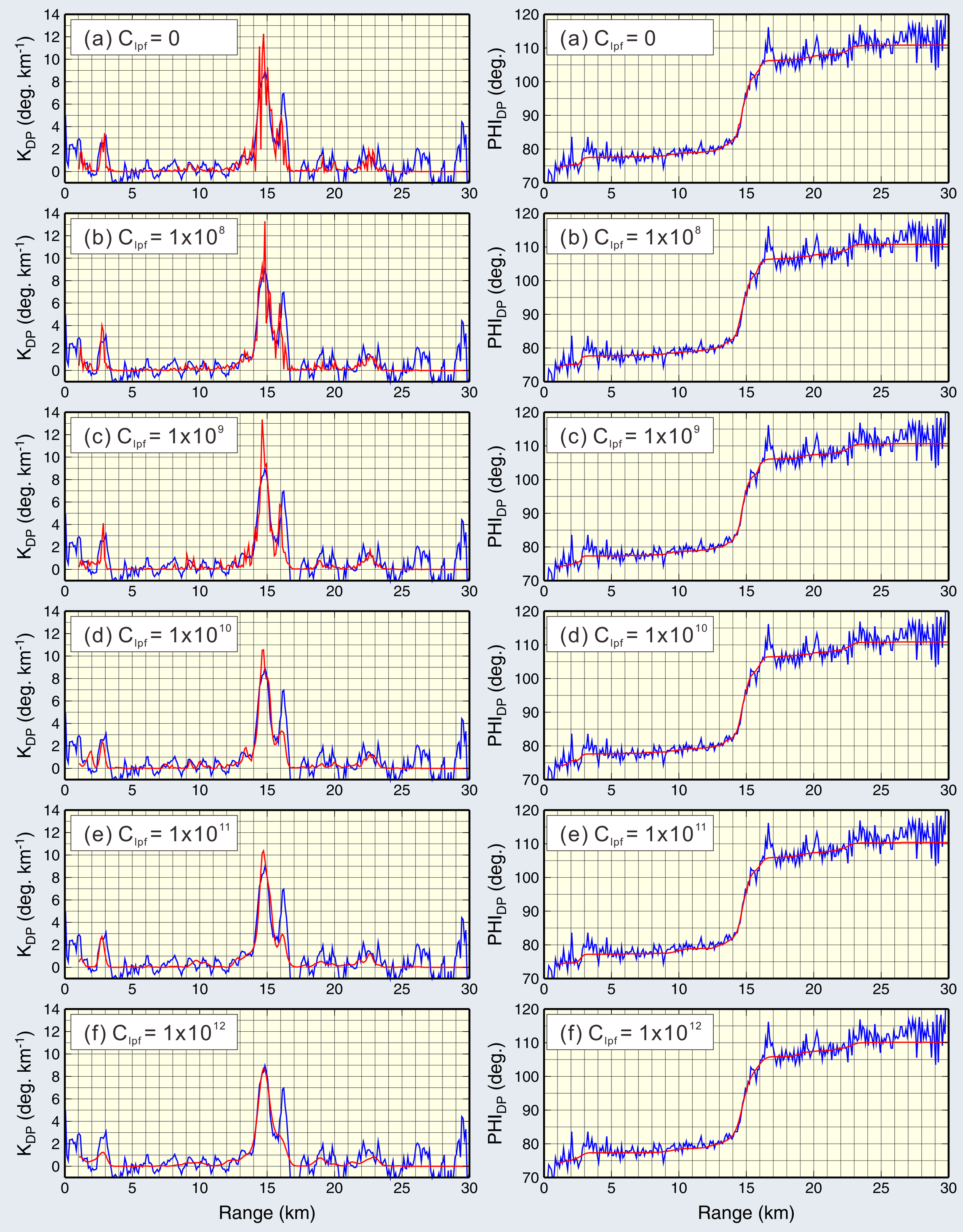



*Schematic range profiles of the observed and analyzed $\Phi_{DP}$ in this method. Horizontal and vertical axes indicate the range from radar and the differential phase, respectively. Dots indicate the observed differential phase ($\Psi$) at the range, and blue solid curve is a final solution of the differential phase in this method. Broken red lines are boundary conditions of the differential phases ($\Phi_{near}$ and $\Phi_{far}$) at the nearest and farthest ranges ($r_{near}$ and $r_{far}$), respectively. Lines LR-N and LR-F are linear regression lines to determine the boundary conditions ($\Phi_{near}$ and $\Phi_{far}$), respectively. $\phi$ and $\psi$ ($\phi'$ and $\psi'$) indicate the difference from the nearest (farthest) boundary condition, respectively.*




*a) The range profiles of the differential phase. Blue profile is observed by NIED's X-band dual-polarimetric radar at Ebina, Japan, at 1402 UTC 9 July 2005. The range gate width of this radar is 100 m. Red profile is analyzed from the observed data by this method ($C_{lpf}$ = $1x10^{10}$).*

*b) The range profiles of the specific differential phase calculated from the observed differential phase shown in (b). Blue profile is estimated by the local linear regression with the regression width of 1 km. Red profile is analyzed by this method ($C_{lpf}$ = $1x10^{10}$).*




*Dependency of the $C_{lpf}$ to the low pass filter in this method. The details of this figures are the same as above. a) $C_{lpf}$ = 0, b) $C_{lpf}$ = $1x10^{8}$, c) $C_{lpf}$ = $1x10^{9}$, d) $C_{lpf}$ = $1x10^{10}$, e) $C_{lpf}$ = $1x10^{11}$, and f) $C_{lpf}$ = $1x10^{12}$. Left and right panels are $K_{DP}$ and $\Phi_{DP}$, respectively.*

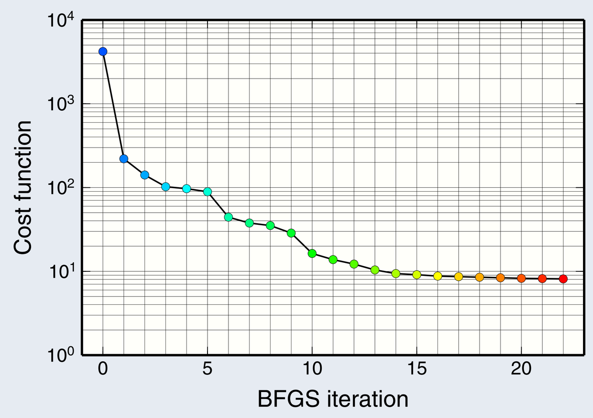
## COMPUTATION COST

To minimize the cost function, we use the Broyden-Fletcher-Goldfarb-Shanno (BFGS) method, which is a kind of numerical optimizations. Because the BFGS method is described by a recurrence formula, a numerical iteration must be performed to suffer from a heavy computation cost. In typical rainfall case, it takes several hundred milliseconds to solve the $K_{DP}$ range-profile of one ray. So it is difficult to embed this method in the radar signal processor.

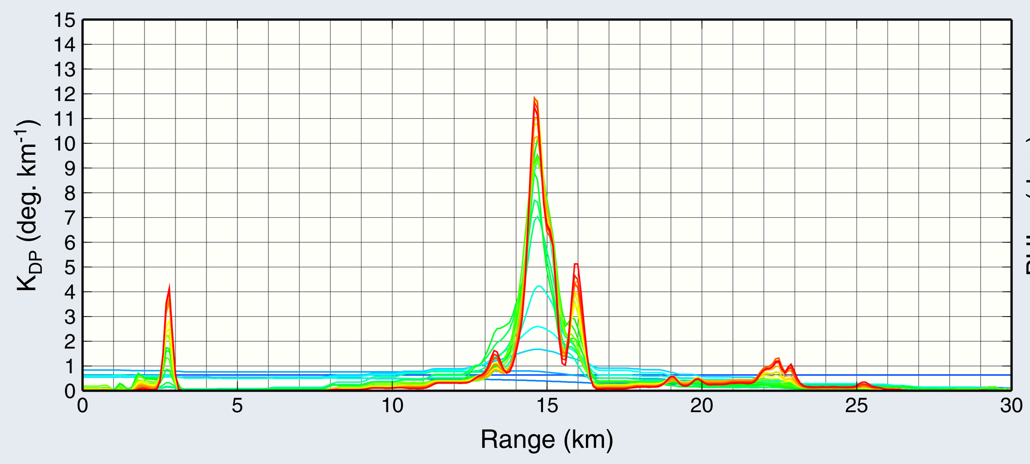
Fortunately, an inexpensive parallel computing is now available. We can calculate the whole $K_{DP}$ in one PPI scan in 10 seconds to 15 seconds. This means that this method is acceptable for the operational use.

**Computation Benchmark Test**

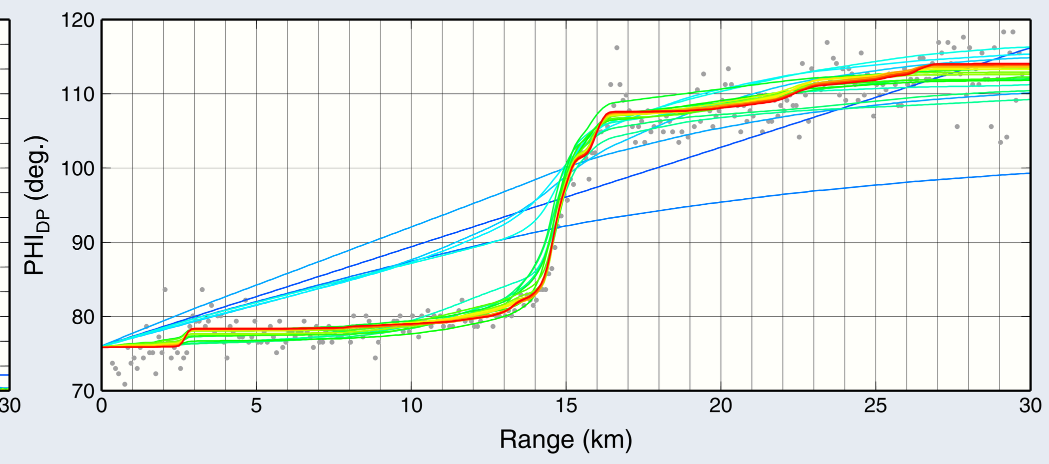
| | |
|---|---|
| **Radar Data** | |
| Radar Frequency | X-band (9375 MHz) |
| Number of range bins / ray | 801 |
| Number of rays / PPI | 360 |
| Max. range | 80 km (100 m x 800) |
| **Computer** | |
| CPU | Intel Xeon E5630 (2.53 GHz, 4 cores) |
| Number of CPUs / node | 2 |
| OS | CentOS 5.5 |
| Communication | 1000Base-T |
| Parallelization | MPI |
| **Calculation Time** | |
| Number of processes | Cal. time (sec.) |
| 1 (1 node) | 97.38 |
| 8 (1 node) | 23.32 |
| 16 (2 node) | 12.38 |
| 24 (3 nodes) | 8.59 |




*A example of the minimizing of the cost function by BFGS method. Colors of plots correspond to the colors of $K_{DP}$ and $\Phi_{DP}$ profiles shown below.*

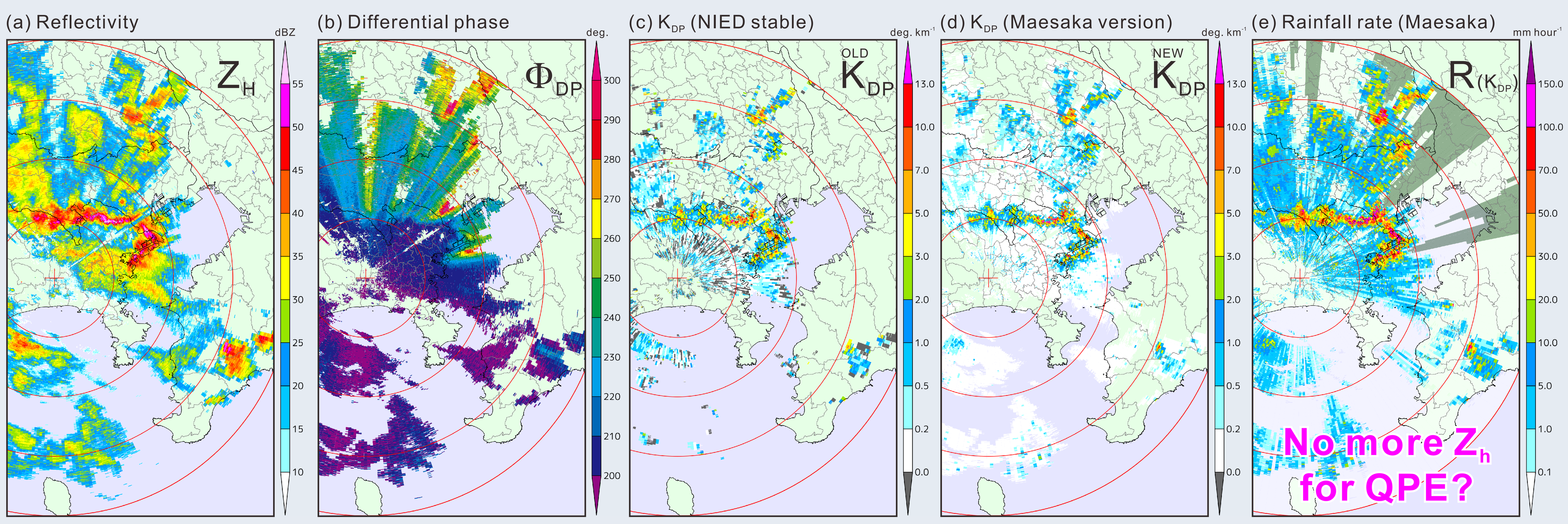



*$K_{DP}$ profile modifications at each BFGS iteration step. Colors of the profiles correspond to the color plots shown in upper right.*



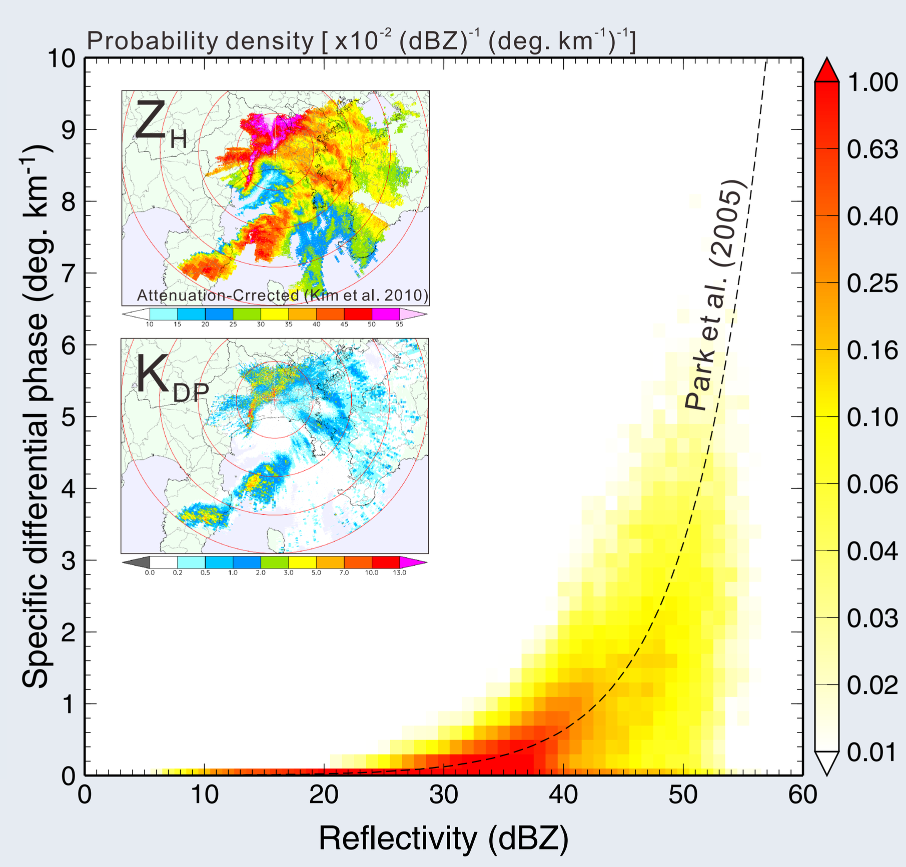

*$\Phi_{DP}$ profile modifications at each BFGS iteration step. Colors of the profiles correspond to the color plots shown in upper right.*

## CASE STUDY (Heavy Precipitation on 10 May 2012)




*PPI images of NIED's X-band polarimetric radar at Ebina city at 0500 UTC 10 May 2012. The elevation angle is 1.2. Range circles are drawn at 20 km intervals from the radar. a) Observed reflectivity (no attenuation correction), b) Observed differential phase, c) Specific differential phase estimated by classic method (NIED stable algorithm: the iterative filter and local linear regression; the data which S/N ratio less than 10 dB are not used.), d) Specific differential phase estimated by this method ($C_{lpf}$ =$1x10^{10}$), and e) Rainfall rate calculated from $K_{DP}$ shown in (f).*

## $Z_H$ VERSUS $K_{DP}$




*Probability density of scatter plots of $Z_h$ versus $K_{DP}$ calculated by this method ($C_{lpf}$ =$1x10^{10}$). The data are observed by NIED's X-band dual-pol. radar at 1402 UTC 9 July 2005.*