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Impacts of small scale rainfall variability in urban areas: a case study with 2D/1D hydrological models in a multifractal framework

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ABSTRACT

In this paper the sensitivity to small scale unmeasured rainfall variability (i.e. scales smaller than 1km in space and 5 min in time, which are usually available with C-band radars) of a 2D/1D model with a 10 m resolution and a semi-distributed 1D model of the same 1.47 km^2 urban area is analyzed. The 2D/1D model is the open source numerical platform Multi-Hydro, which couples (open source) distributed models of involved hydrological/hydraulic processes and is currently being developed at Ecole des Ponts ParisTech. The methodology implemented to evaluate the uncertainties consists in generating an ensemble of realistic rainfall fields downscaled to a resolution of 12.3 m in space and 18.75 s in time with the help of a stochastic universal multifractal model. The corresponding ensemble of hydrographs is then simulated. It appears that the uncertainty is significant (for example the upper tail of the probability distribution of the peak flow distribution exhibits a power-law distribution) and that Multi-Hydro unveils much more uncertainty than the simpler 1D model. This points out a need to develop high resolution distributed modelling in urban areas.

KEYWORDS

Rainfall variability, 2D/1D modelling, multifractals, space-time downscaling

1 INTRODUCTION

Rainfall variability has a significant impact on river discharges (see Singh, 1997 for a review). This impact is enhanced in urban areas where the response times of catchments are shorter and the coefficient of imperviousness are larger meaning that a significant fraction of the rainfall is immediately active (Aronica and Cannarozzo, 2000; Segond et al., 2007). The under-representation of rainfall variability in input data of models affects the confidence one should have in its predictions. A better understanding of rainfall variability in urban areas and its impact on simulated flow is needed both theoretically and operationally. Indeed Real Time Control (RTC, see Schütze et al., 2004 for a review of its rapid development over the last decades) of sewer networks, which aims at reducing urban flooding and pollution, relies on the use of such models.

In recent papers Gires et al. (2011b, 2012a, 2012b) quantified the impact of small scale unmeasured rainfall variability (i.e. at scales smaller than the C-band radar resolution of 1km x 1km x 5min, which is usually provided by national meteorological services of Western Europe countries) on urban discharges simulated with the help of semi-distributed urban hydrological / hydraulic 1D models. Two urban areas were studied: a 3400 ha one located near Paris and a 900 ha one located in the North of London. The methodology implemented relies on the generation and analysis of realistic ensembles: (i) generation of an ensemble of realistic rainfall fields through a stochastic multifractal downscaling of the radar data, (ii) Simulation of the corresponding ensemble of hydrographs with a semi-distributed 1D model, (iii) Quantification of the variability among these ensembles. A limitation of these works was that the size of the sub-catchments (roughly 17 ha on average), which are considered as homogenous objects, did not enable to fully grasp the actual rainfall spatial variability. In this paper we implement the same methodology on a portion of size 147 ha of the previous Paris area case study (see Figure 1). Two types of model are used: the same semi-distributed operational one and a 2D/1 D fully distributed one called Multi-Hydro. It is a numerical platform currently being developed at Ecole des Ponts ParisTech and validated in the framework of FP 7 SMARTeST European Project (v1 El Tabach et al., 2009; v2 Giangola-Murzin et al.,2012).

The studied rainfall event and the downscaling technique implemented are presented in section 2.1. In section 2.2 there is a brief presentation of Multi-Hydro and some associated modelling issues. The 147 ha studied urban area and its representation with the two models is presented in section 2.3. Results are discussed in section 3.

Figure 1: Picture of the 147 ha studied area, located in the city of Sevran (North-East of Paris)

2 DATA AND METHODS

2.1 Rainfall data and downscaling technique

Only a short description of the rainfall data and the downscaling technique will be presented here, because both have been extensively described in Gires et al. 2012a, 2012b. In this paper which is a preliminary analysis, only one rainfall event is used. It occurred over the Paris area on February 9th, 2009. The data, whose resolution is 1 km in space and 5 min in time, comes from the C-band radar of Trappes which is operated by Météo-France. The rainfall rate R is computed from the radar reflectivity Z with the help of a standard $Z=aR^b$ relationship with a=200 and b=1.6 (Z in $mm^6.m^{-3}$ and R in $mm.h^{-1}$)

Figure 2.a displays the estimated total rainfall depth (in mm) observed during the 13 h of this event, and over a square area of size 256 km x 256 km centred on the radar. The studied catchment is located at approximately 45 km of the C-band radar meaning that the rainfall estimates are still reliable. Over the studied catchment (black box in Figure 2.a) the event lasted roughly 4 hours and the average rainfall depth is of 15 mm. This roughly corresponds to a 5 month return period event for this area.

The downscaling technique relies on the very convenient framework of Universal Multifractals (Schertzer and Lovejoy, 1987), which has been extensively used (see Lovejoy and Schertzer, 2007 for a recent review, de Lima, 2009, Verrier, 2010 for applications in hydrology) to analyse and simulate geophysical fields extremely variable over wide range of scales (Schertzer and Lovejoy 2011 for a recent review). Indeed it assumes that rainfall is generated through a space-time cascade process. In the discrete case, which is implemented here, the rainfall intensity over a large scale structure is distributed in space and time step by step. At each step the "parent structure" is divided into several "child structures". To be consistent with the scaling of life-time vs. the structure size in the framework of the Kolomogorov picture of turbulence (Kolmogorov, 1962) the scale of the structure is divided by 3 in space and 2 in time at each step of the cascade process (Marsan et al., 1996; Biaou et al., 2005; Gires et al., 2011), which leads to 18 child structures. See Figure 2.b for an illustration of two steps of such a process. The value affected to the child structure is equal to the one of the parent structure multiplied by a random multiplicative increment $\mu\varepsilon$. In the specific framework of Universal Multifractals, the probability distribution of $\mu\varepsilon$ is fully determined with the help of only two parameters which are estimated over the available range of scales (1 – 256 km). In other words, the downscaling implemented in this paper simply consists in stochastically continuing the cascade process that is assessed over the available data. Four steps of the process are implemented which enables to simulate realistic rainfall fields of resolution 12.3 m in space and 18.75 s in time. The cascade process is conservative on average. More details about the multifractal analysis of the rainfall event and its downscaling can be found in Gires et al. 2012b. More details about the simulation of Universal Multifractal fields can be found in Pecknold et al. 1993 and Lovejoy and Schertzer 2010.

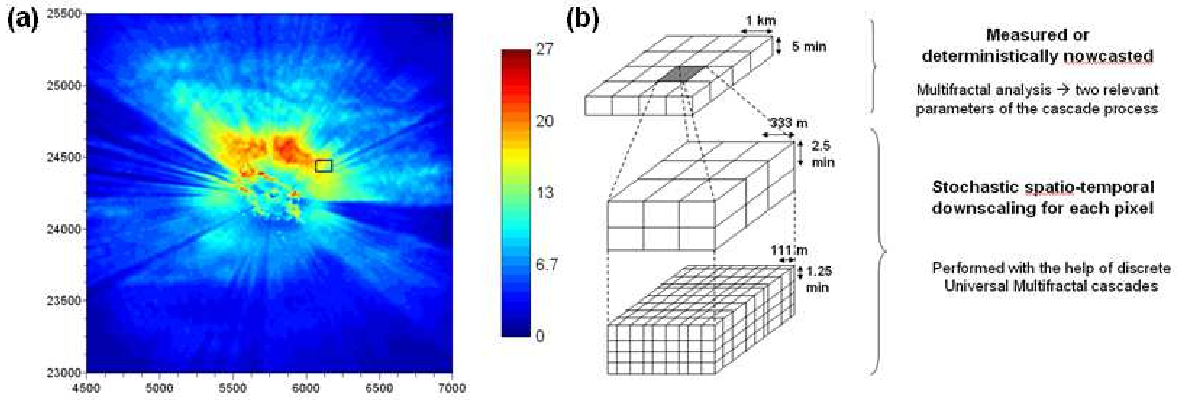

Figure 2: (a) Map of the total rainfall depth (in mm) during the rainfall event of February 9th, 2009. The area is of size 256 km x 256 km, and the coordinate system is the "extended Lambert II" system (unit = hm). The Meteo-France C-band radar of Trappes is located in the centre of the image. The studied catchment is located in the South of the black box. (b) Illustration of two steps of the cascade process implemented to downscale the rainfall data.

2.2 Short presentation of Multi-Hydro

The second version of Multi-Hydro that is used in this paper consists in an interactive coupling between a 2D model representing surface runoff and infiltration (TREX, Two dimensional Runoff, Erosion and eXport model, Velleux et al., 2011) and a 1D model of sewer networks (SWMM, Storm Water Management Model, Rossman, 2007). Only the hydraulic part of SWMM is used to model water flow in pipes, and not the hydrologic one. The main input data is a precise description of the sewer network, the topography, and the land use distribution. In this case study five different classes of land use are used (forest, grass, roads, building and gullies), each being fully characterized by its hydraulic conductivity (m/s), capillary suction (m), moisture deficit (no unit, ranging from 0 to 1), Manning coefficient (unitless) and depth of interception (mm). The interactions between the sewer system and surface flow are handled through the gullies where water can circulate in both ways, i.e. from surface to sewer in standard situation, and the other way in case of sewer overload. More details about Multi-Hydro can be found in Giangola-Murzyn et al. (2012) paper for this conference.

In this paper, more details will be provided only concerning the elevation data, to stress the need for carefully choosing the resolution of the model. Indeed the digital terrain model provided by the Institut National Géographique, does not take into account anthropogenic elevation modification. As a consequence, there is an option in Multi-Hydro that decreases by 15 cm the elevation of the road pixels and that increases the building ones by 5 m to prevent water from running through these pixels. The rainfall collected by the building pixels is directly routed to the nearest gully. Implemented with a 1 m resolution (i.e. the size of the pixels is 1m), this way of representing buildings is rather correct. With lower resolution, some difficulties might arise. For example Figure 3 displays the land use cover and modified elevation with a 10 m resolution, for an area located in the East of the Seine-Saint-Denis County, not far from the studied catchment. It appears that with this resolution the water running from the North-East of the catchment has no exit way to reach the outlet and remains trapped in the pixel highlighted in red. With a 1 m resolution the water can run along the West side of the building. This means that representing the hydraulic behaviour of the house pixels as previously explained might lead to errors with too low resolutions. A possibility, which will be tested in the near future, consists in changing the way the building pixels are modelled with the resolution. For example their elevation might not be changed, so that surface water can run through them, but with an increased Manning coefficient to represent the fact that actually water cannot run through the whole pixel but only along the sides of the buildings. The rainfall collected by these pixels would still be directly rooted to the nearest gully. Finally, let us mention that this difficulty does not appear in the catchment studied in this paper. The aim of this discussion was only to stress the importance of carefully choosing both the resolution and the modelling, and the fact that both are necessarily related.

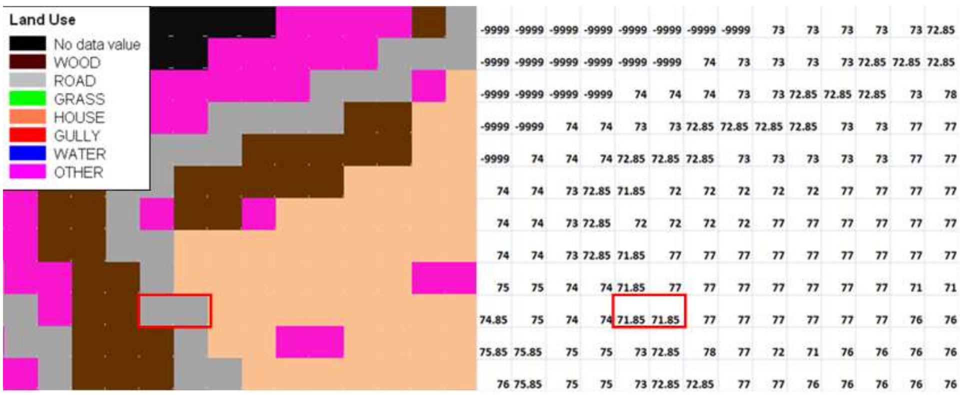

Figure 3: Land use distribution (left) and modified elevation in m (right) of a catchment of Seine-Saint-Denis

2.3 Studied area and its representation with the help of two models

The studied catchment is a 1.47 km^2 urban area located in the city of Sevran (Seine-Saint-Denis county, North-East of Paris). This area is known for regular sewer overflows, and there is a project to build a stormwater storage basin to limit them and also to limit water transfer during heavy rainfall to the downstream area just North of it because it suffers frequent pluvial flooding. The catchment is modelled with the help of Multi-Hydro with a 10 m resolution. Figure 4.a displays the corresponding land use distribution and the sewer system. In that case the land use class “other” mainly reflects the small gardens nearby houses and the large un-built area which is a former Kodak factory, and is therefore considered as “grass”. With this resolution, the road network is clearly visible, but the importance of houses might be overestimated with regards to their attending small garden.

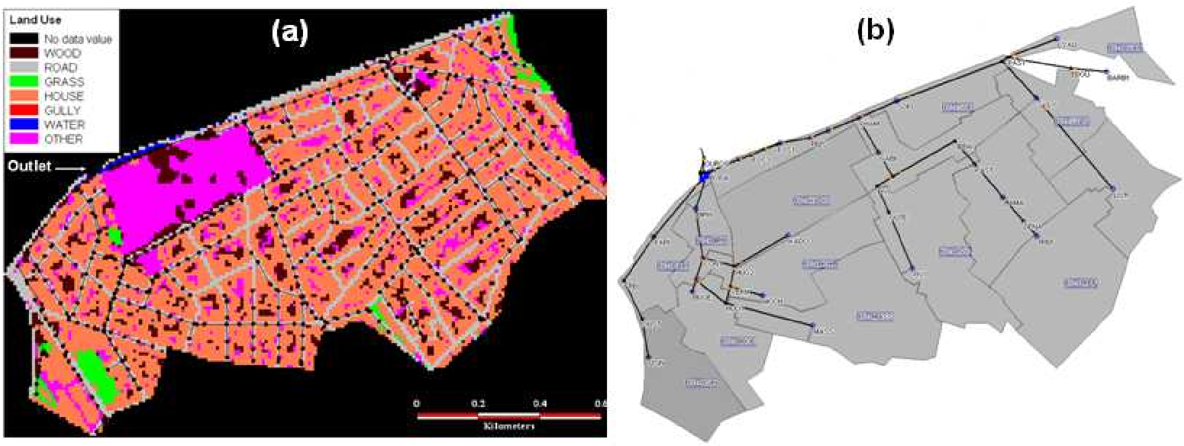

Figure 4: (a) Map of the land use distribution with a 10 m resolution used in Multi-Hydro. The sewer network modelled with SWMM is superposed to this map (the nodes, which are either a manhole or a gully, and conduits are visible along most of the roads). (b) Snapshot of the representation of this area with the 1D model Canoe. The sub-catchments and the modelled sewer network are visible.

The Direction Eau et Assainissement of Seine-Saint-Denis (the local authority in charge of urban drainage) calibrated and runs operationally the semi-distributed 1D model Canoe (Allison, 2005) on this area. In Canoe the hydrologic response of each sub-catchment is modelled by a lumped model (a linear reservoir) and the flow in the pipes is modelled with the help of a numerical solution of Saint-Venant equations. The studied area (see Figure 4.b) is divided into 16 sub-catchments whose size is ranging from 4 to 14.5 ha..Finally let us mention that the total area studied with the 1D model is 1.39 km^2. The difference with the area modelled with Multi-Hydro corresponds to the white area in the upper-right corner of Figure 4.b, which is not included in the 1D model. The average coefficient of imperviousness is equal to 53% in the 1D model, whereas it is of 76% (obtained by considering the pixels whose land use class is “road”, “building” or “gully”) with the land use distribution used in Multi-Hydro. This difference is quite great and is likely to be due to the resolution of Multi-Hydro. Indeed with a higher resolution, this coefficient would be smaller because the land use “building” is priority over the land use “forest” or “grass (meaning that a pixel containing a portion of garden and one of building is considered as “building”). A higher resolution was not implemented for now (but will be in near future) because the computation time would be too great, especially for ensemble simulations. The sewer network is much more detailed in Multi-Hydro.

3 RESULTS AND DISCUSSION

Let us first study the temporal evolution of the simulated flow with the raw radar data at the outlet. It is displayed for both models in Figure 5. It should be emphasised that this rainfall event does not generate any sewer overflow. Both curves exhibit rather comparable patterns but with quite different numerical values. More quantitatively the Nash-Sutcliff coefficient is equal to -0.18 with the 1D model as a reference and to 0.64 with Multi-hydro as a reference. There is no significant time shift between the two models. The differences are mainly due the coefficient of imperviousness which is equal to 76 % and 53% respectively the Multi-Hydro and the 1D model. Implementing Multi-Hydro with a higher resolution and comparing with actual flow measurement would be needed to determine the most accurate model.

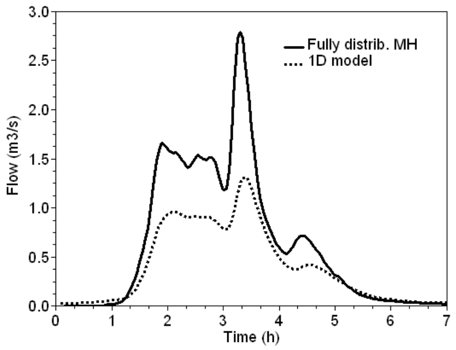

Figure 5: Comparison of simulated flow at the outlet for both models

In order to quantify the uncertainty associated with small scale unmeasured rainfall variability for the Multi-Hydro model, the following methodology is implemented: (i) An ensemble of 100 realistic downscaled rainfall fields with a resolution of 12.3 m in space and 18.75 s in time is generated by implementing 4 steps of spatio-temporal cascades to the radar data. (ii) The corresponding ensemble of hydrographs is then simulated. (iii) The variability among the hydrographs is characterized with the help of the envelop curves $Q_{0.1}$, $Q_{0.5}$ and $Q_{0.9}$, which are respectively made of the 10, 50 and 90% quantiles (in m^3/s) estimated for each time step. Figure 6.a displays the flow simulated with raw radar data (Q_{radar}), $Q_{0.1}$, $Q_{0.5}$ and $Q_{0.9}$ for the outlet of the catchment. Before going on, it should be mentioned that the observed differences between the hydrographs are not due to variations in the total rainfall amount, but to variations in the spatio-temporal distribution of rainfall. Figure 7 displays the temporal evolution of the average rainfall over the studied area for the 100 generated samples (in black) and for the raw radar data (in red). The disparities are much smaller than the one observed on the simulated discharges. The raw radar total rainfall amount is of 15.2 mm, whereas it is of 15.2 ± 0.12 mm (coefficient of variation of 0.8%) for the generated downscaled rainfall fields. The curves of Figure 6.a should be compared with similar ones obtained with the 1D model (see Figure 6.b), where given the size of the homogeneous sub-catchments the rainfall was downscaled only to 111 m in space and 1.25 min in time. The difference between $Q_{0.1}$ and $Q_{0.9}$ is much greater for Multi_Hydro than for the 1D model indicating that it enables to unveil much more uncertainty during the whole event and not only during the peak flow. This means that the results provided by the 1D-model are simply falsely reassuring and that there is a clear need for taking into account small scale phenomenon in urban hydrology.

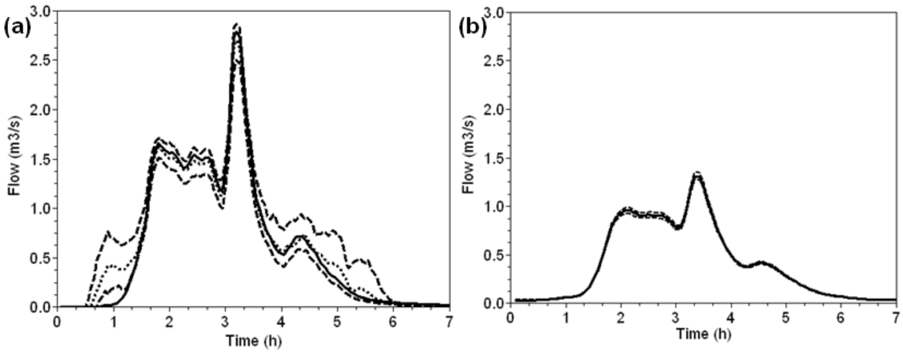

Figure 6: Hydrographs $Q_{0.9}$ (long dash), $Q_{0.5}$ (dot dash), $Q_{0.1}$ (long dash) and Q_{radar} (solid) for the outlet of the studied catchment with Multi-Hydro (a) and the 1D-model (b)

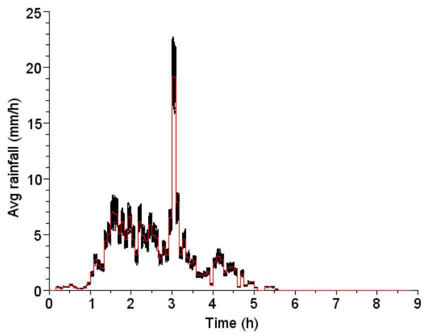

Figure 7: Temporal evolution of average rain rate over the studied catchment for the 100 downscaled (to a resolution of 12.3 m in space and 18.75 s in time) rainfall fields (in black) and the raw radar data (in red)

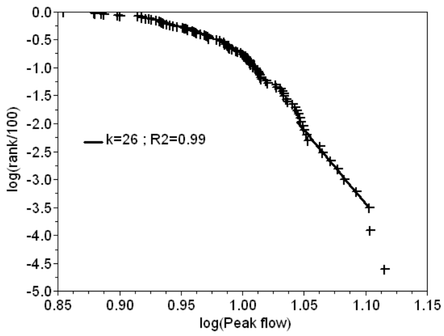

Figure 8: Determination curve of the power law exponent k for peak flows for the outlet with Multi-Hydro

Two indicators are used to analyse more quantitatively the peak flow distribution. First its middle portion is characterized with the help of a pseudo coefficient of variations defined with the help of the quantiles:

$$CV'_{(1)} = \frac{Q_{0.9}(t_{PF,radar}) - Q_{0.1}(t_{PF,radar})}{2 * PF_{radar}} \quad (1)$$

Where $t_{PF,radar}$ is the time of occurrence of the peak flow with the raw radar data. Second the extreme values of this distribution are rather well characterized with the help of a power law distribution, as it often occurs with multifractals:

$$\Pr(X_{\max} > x) \approx x^{-k} \quad (2)$$

Where k is an exponent that defines the strength of the fall-off. A greater k corresponds to a lower dispersion in the ensemble of peak flows. Figure 8 displays Eq. 2 for the outlet of the studied catchment with Multi-Hydro. The coefficient of determination is great enough (0.99) to validate the use of a power-law. Concerning the numerical values for the outlet *CV*' is equal to 6.0% and 7.5%, and k is equal to 34 and 26 for respectively the 1D model and Multi-Hydro. This numerically confirms, especially for k, the qualitative comments of the previous paragraph.

4 CONCLUSION

Universal multifractals are used to quantify the uncertainty associated with small scale unmeasured (i.e. occurring at scales smaller than 1 km in space and 5 min in time) rainfall variability on the outputs of Multi-Hydro, a newly developed fully distributed urban hydrologic/hydraulic numerical platform, and a standard semi-distributed 1D model implemented on the same 1.47 km^2 urban area located in Sevran, near Paris (France). The methodology basically consists in generating an ensemble of realistic downscaled rainfall fields and simulating the corresponding ensemble of hydrographs. This enables to quantify the uncertainty. It appears that the uncertainty is significant and cannot be neglected. Furthermore the Multi-Hydro model unveils much more uncertainty not only during the peak flow, but during the whole event, i.e. for moderate rain rates. The latter was not expected and if confirmed would require small scale phenomenon to be taken into account much more carefully in urban hydrology and not only for flood management. This needs to be confirmed by similar analysis with more rainfall events and also on other catchments. This also points out that in terms of modelling the use of fully distributed models should be developed. In terms of rainfall, there is a need for higher resolution data in urban areas. To achieve this, the use of X-band radars which provide hectometric resolution would be highly beneficial. Further investigations with heavier rainfall events that generate urban pluvial flooding should also be performed to confirm this need for high resolution modelling.

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